CHAOS SYNCHRONIZATION IN A MULTI-TROPHIC ECOLOGICAL SYSTEM BY ACTIVE CONTROL WITH UNCERTAIN PARAMETERS

[1]YILMAZ UYAROGLU, [2]UGUR ERKIN KOCAMAZ, [3]GULTEKIN CAGIL

[1]Electrical & Electronics Engineering Department, Engineering Faculty, Sakarya University, 54187 Serdivan, Sakarya, Turkey
[2]Computer Technologies Department, Karacabey Vocational School, Uludağ University, 16700 Karacabey, Bursa, Turkey
[3]Industrial Engineering Department, Engineering Faculty, Sakarya University, 54187 Serdivan, Sakarya, Turkey
E-mails: [1]uyaroglu@sakarya.edu.tr, [2]ugurkocamaz@gmail.com, [3]cagil@sakarya.edu.tr

Abstract- In this study, based on the mathematical model of a nonlinear three-dimensional multi-trophic ecological model, chaos synchronizationis investigated by using the active control technique with uncertain parameters. Using the active control theory, the synchronization errorsare ensured to zero. Designed controllers are applied to the chaotic multi-trophic ecological modelsthat have different parameter values and initial conditions. After controllers are activated, the change in multi-trophic ecological models from one chaotic behavior to another chaotic behavior is demonstrated with numerical simulations. Simulation results also show that the proposed techniqueis effective for the synchronization of chaotic multi-trophic ecological systems which have different parameter values.

Keywords- Multi-Trophic Ecological System, Chaos Synchronization, Active Control, Uncertain Parameters.

I. INTRODUCTION

Chaotic systems are very sensitive to theirparameter values and initial conditions. Furthermore, they have non-periodic fluctuations.Since the first attractor was found by Lorenz from a simplified model of convection in the Earth's atmosphere in 1963 [1], many scientists have extensively studied the existence of chaos in nature. Some chaotic systems were discoveredin a variety of fields including physics [2], chemistry [3], ecology [4], biology [5], finance [6], etc.

Sometimes, the synchronization of systems is needed. However, it can be a difficult task because of the chaotic behavior.So, the chaos synchronization has also been receiving increasingly attentions from researchers.After Pecora and Carroll presented the pioneer chaos synchronizationstudy in 1990 [7], many effective control methods have been applied for the synchronization of chaotic systems such as active control [8–11], passive control [12], sliding mode control [12] and adaptive control [13]. Due to its simplicity and efficiency, the active controltechnique has become one of the common chaos synchronization methods.

In this study, the synchronization of the chaotic multi-trophic ecological model is investigated based on the active controltechnique. The non-periodic fluctuations may result in unpredictable and unwanted trajectories.The synchronization of species in food chainmodels can have economic advantages. So, the synchronization of chaotic motions in the multi-trophic ecological systems has great importance.

The rest sections of this paper is organized as follows: In Section 2, the chaotic multi-trophic ecological model is described. In Section 3, the active controllers are constructed and the synchronization of chaotic multi-trophic ecological systems having different parameter values is achieved. In Section 4, numerical results are given and discussed graphically. Simulation results show that the designed controllers can regulate the synchronization of chaotic multi-trophic ecological systems effectively.Finally, conclusions are given in Section 5.

II. CHAOTIC MULTI-TROPHIC ECOLOGICAL SYSTEM

The multi-trophic ecological system model which shows chaotic behavior is firstly studied and analyzed by Hastings and Powell in 1991 [14]. The state equations of chaotic multi-trophic ecological system model are written as follows [14]:

$$\begin{cases} \dot{x} = rx(1-kx) - f_1(x)y, \\ \dot{y} = -d_1 y + f_1(x)y - f_2(y)z, \\ \dot{z} = -d_2 z + f_2(y)z, \end{cases} \tag{1}$$

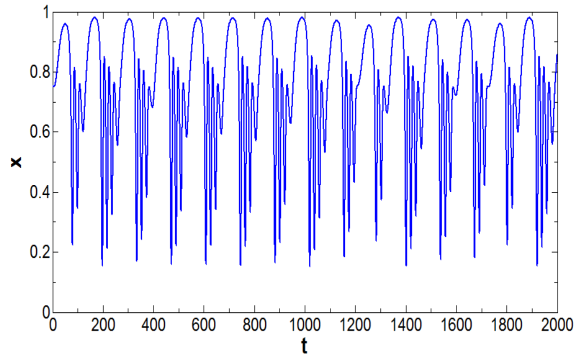
with

$$f_i(u) = a_i u/(1+b_i u), \tag{2}$$

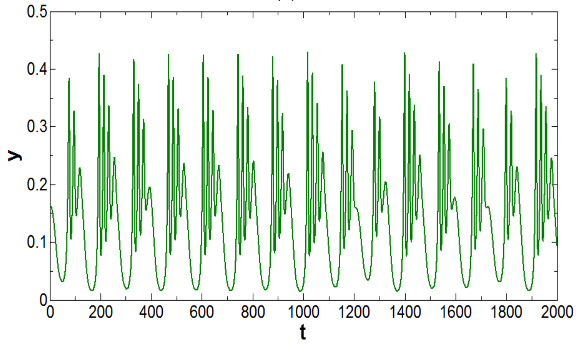
where x is the numbers of the species at the lowest level of the food chain, y is the number of the species which preys upon x, z is the numbers of the species which preys upon y,r is the intrinsic growth rate, k is the carrying capacity of species x, d_1 and d_2 are constant death rates for species y and z, respectively [14]. The constants a_i and b_i for i = 1, 2 parametrize the saturating functional response; b_i is the prey population level where the predation rate per unit is half its maximum value [14].

Hastings and Powell(1991) demonstrated bifurcation diagrams of the model system (1) and (2) for parameter values r = 1, k = 1, a_1 = 5, a_2 = 0.1,b_2 = 2, d_1 = 0.4 and d_2 = 0.01 with b_1 is varied from 2.2 to 6.2, and showed that the multi-trophic ecological system has chaotic behaviors [14].For these values and b_1 = 3 with initial values x(0) = 0.75, y(0) =

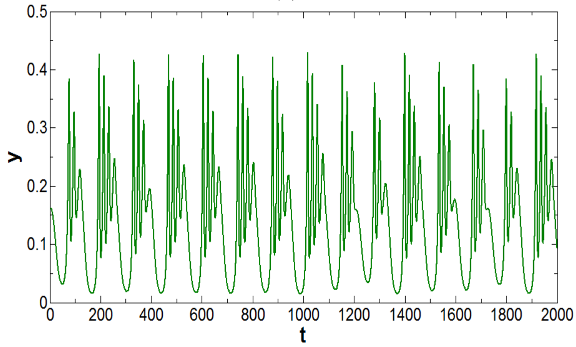
0.16and $z(0) = 9.9$, the chaotic time series, phase portraits and 3D graph of multi-trophic ecological system are obtained by using the equations of system (1) and (2) in Matlab and demonstrated in Fig. 1, Fig. 2 and Fig. 3, respectively.Fourth-order Runge–Kutta method with fixed-time step $10^{-3}$is used as integral solver function in the simulations.



(a)

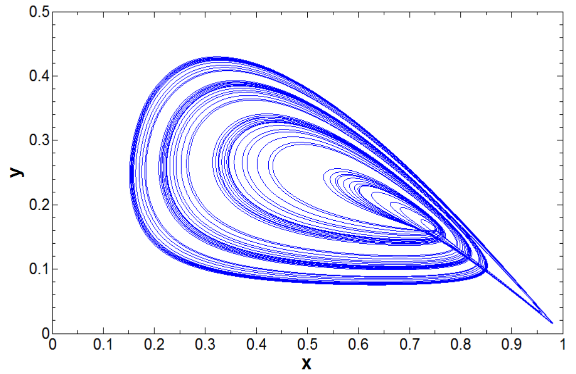


(b)

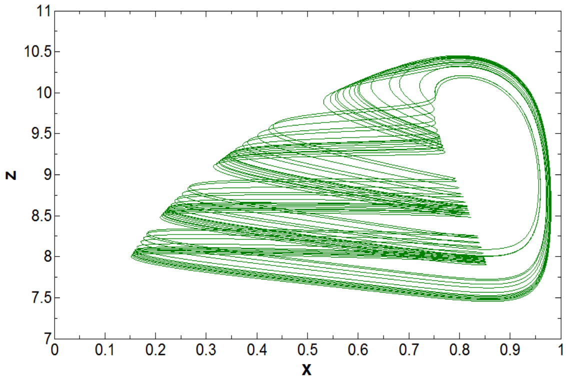


(b)

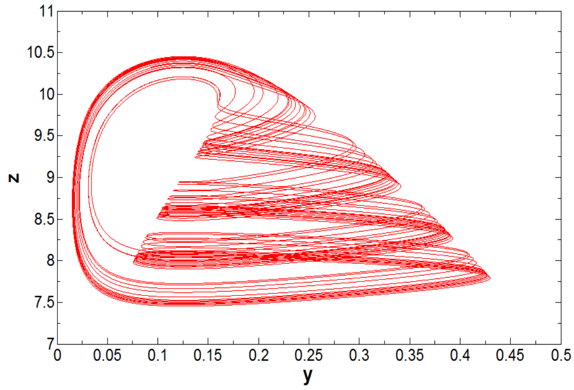
Fig. 1. Multi-trophic ecological system with $r = 1$, $k = 1$, $a_1 = 5$,$a_2 = 0.1$,$b_1 = 3$, $b_2 = 2$, $d_1 = 0.4$ and $d_2 = 0.01$in (a) x, (b) y, and (c) z time series.



(a)



(b)



(c)

Fig. 2. Projection of multi-trophic ecological attractor with $r = 1$,$k = 1$, $a_1 = 5$, $a_2 = 0.1$,$b_1 = 3$, $b_2 = 2$, $d_1 = 0.4$ and $d_2 = 0.01$ into the (a) x–y, (b) x–z, and (c) y–zphaseportraits.

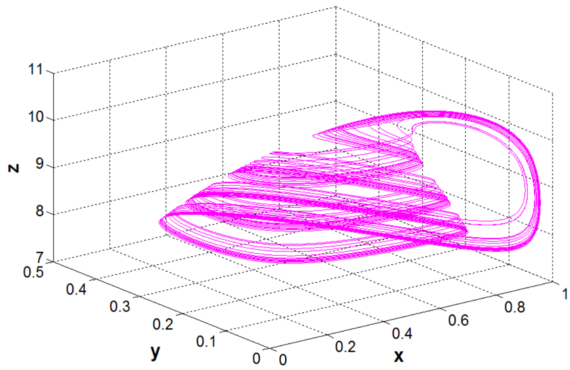


Fig. 3. Projection of multi-trophic ecological attractor with $r = 1$,$k = 1$, $a_1 = 5$, $a_2 = 0.1$,$b_1 = 3$, $b_2 = 2$, $d_1 = 0.4$ and $d_2 = 0.01$into the 3D phase plane.

In 2007, Stone and He proposedanother chaotic parameter values of the multi-trophic ecological system (1)and (2) with demonstrating the time seriesand y–z phase portrait[15].Complicated dynamic behavior of modified Hastings–Powell equations were also investigated by Stone and He [15]. For parameter values $r = 2.5$, $k = 1.5$, $a_1 = 4$,$a_2 = 4$, $b_1 = 3$, $b_2 = 3$,$d_1 = 0.4$ and $d_2 = 0.6$ with initial values $x(0) = 0.6$, $y(0) = 0.2$ and $z(0) = 0.08$, the chaotic time series, phase portraits and 3D graph of multi-trophic ecological system are demonstrated in Fig. 4, Fig. 5 and Fig. 6, respectively.

The chaos synchronization in a nonlinear ecological system model whose chaotic behaviorswere shown by references [14] and [15] is investigated in this study.

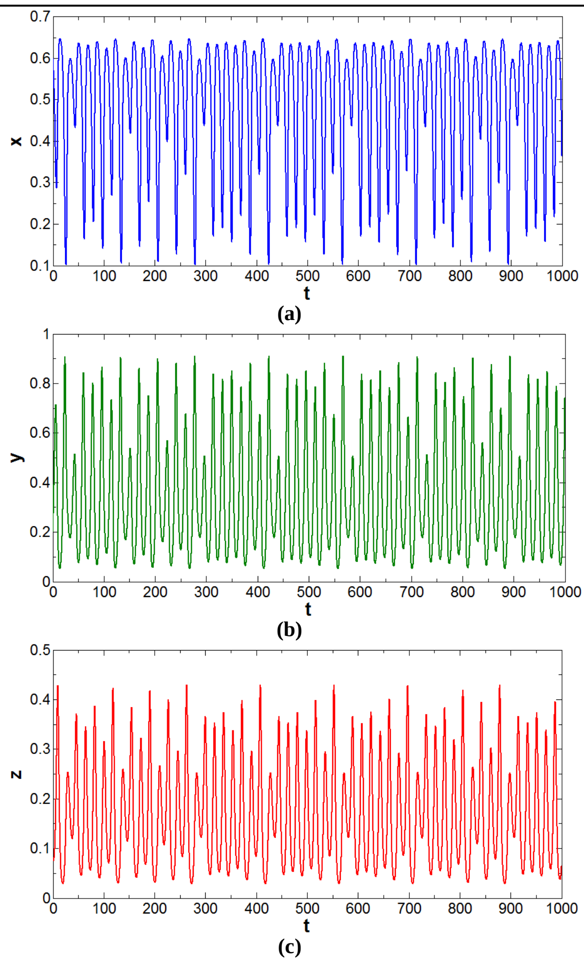
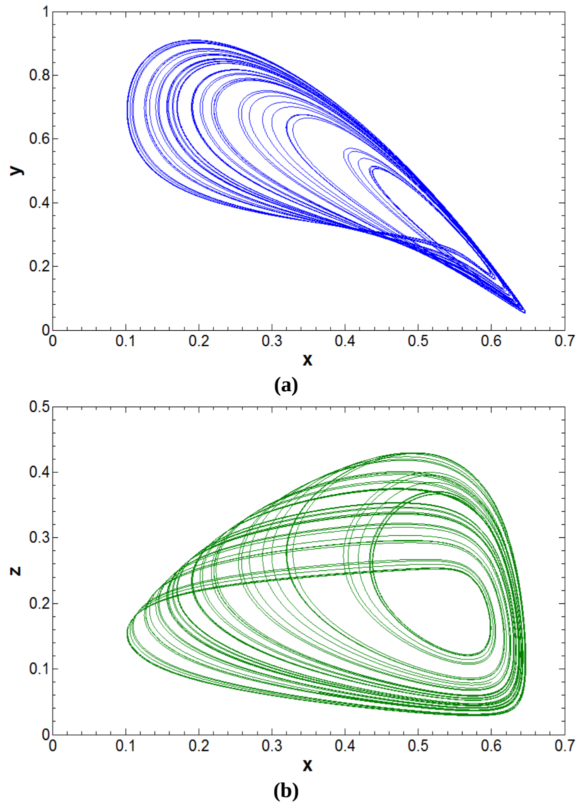


Fig. 4. Multi-trophic ecological system withr = 2.5, k = 1.5, a_1 = 4, a_2 = 4, b_1 = 3, b_2 = 3, d_1 = 0.4 and d_2 = 0.6in (a) x, (b) y, and (c) z time series.



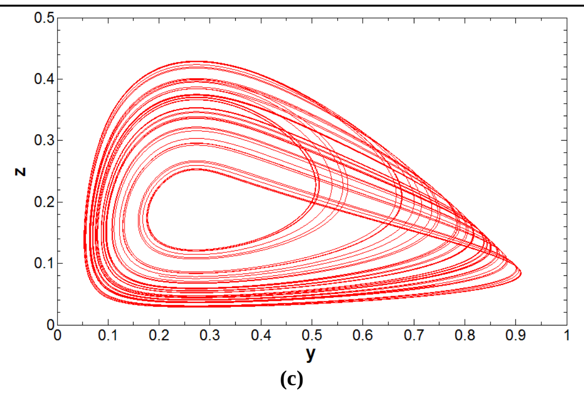


Fig. 5. Projection of multi-trophic ecological attractor withr = 2.5, k = 1.5, a_1 = 4, a_2 = 4, b_1 = 3, b_2 = 3, d_1 = 0.4 and d_2 = 0.6into the (a) x–y, (b) x–z, and (c) y–z phase portraits.

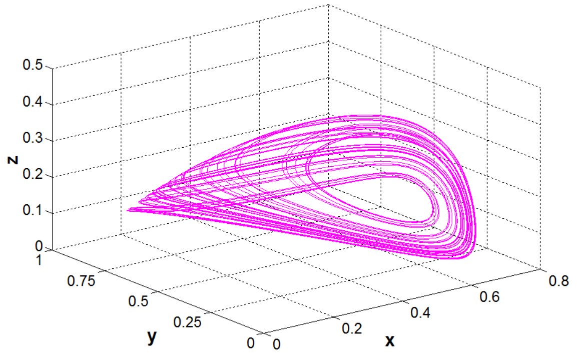


Fig. 6. Projection of multi-trophic ecological attractor withr = 2.5, k = 1.5, a_1 = 4, a_2 = 4, b_1 = 3, b_2 = 3, d_1 = 0.4 and d_2 = 0.6into the 3D phase plane.

III. SYNCHRONIZATION OF CHAOTIC MULTI-TROPHIC ECOLOGICAL SYSTEM

In this section, the synchronization of a chaotic multi-trophic ecological system with uncertain parameters is achieved using active control theory. In order to apply the synchronization, two chaotic multi-trophic ecological systems that have different parameter and initial values are considered where the drive system is denoted by the subscript 1, and the response system is denoted by the subscript 2. They are respectively given as

$$\begin{cases} \dot{x}_1 = rx_1(1-kx_1) - a_1x_1y_1/(1+b_1x_1), \\ \dot{y}_1 = -d_1y_1 + a_1x_1y_1/(1+b_1x_1) \\ \qquad -a_2y_1z_1/(1+b_2y_1), \\ \dot{z}_1 = -d_2z_1 + a_2y_1z_1/(1+b_2y_1), \end{cases} \tag{3}$$

and

$$\begin{cases} \dot{x}_2 = \bar{r}x_2(1-\bar{k}x_2) - \bar{a}_1x_2y_2/(1+\bar{b}_1x_2) + u_1, \\ \dot{y}_2 = -\bar{d}_1y_2 + \bar{a}_1x_2y_2/(1+\bar{b}_1x_2) \\ \qquad -\bar{a}_2y_2z_2/(1+\bar{b}_2y_2) + u_2, \\ \dot{z}_2 = -\bar{d}_2z_2 + \bar{a}_2y_2z_2/(1+\bar{b}_2y_2) + u_3, \end{cases} \tag{4}$$

where u_1, u_2 and u_3 in system (4) are external active control inputs. The synchronization error is defined as $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$ and $e_3 = z_2 - z_1$. Then, the error dynamics are obtained as

$$\begin{cases} \dot{e}_1 = \bar{r}e_1 + (\bar{r} - r)x_1 - \bar{r}\bar{k}x_2^2 + rkx_1^2 \\ \qquad - \bar{a}_1 x_2 y_2/(1+\bar{b}_1 x_2) + a_1 x_1 y_1/(1+b_1 x_1) + u_1, \\ \dot{e}_2 = -\bar{d}_1 e_2 + (d_1 - \bar{d}_1)y_1 + \bar{a}_1 x_2 y_2/(1+\bar{b}_1 x_2) \\ \qquad - a_1 x_1 y_1/(1+b_1 x_1) - \bar{a}_2 y_2 z_2/(1+\bar{b}_2 y_2) \\ \qquad + a_2 y_1 z_1/(1+b_2 y_1) + u_2, \\ \dot{e}_3 = -\bar{d}_2 e_3 + (d_2 - \bar{d}_2)z_1 + \bar{a}_2 y_2 z_2/(1+\bar{b}_2 y_2) \\ \qquad - a_2 y_1 z_1/(1+b_2 y_1) + u_3, \end{cases} \quad (5)$$

The purpose of synchronization is to ensure the error system (5) asymptotically stable at the origin. The nonlinear terms in system (5) can be eliminated by defining the active control functions u_1, u_2 and $u_3$as follows:

$$\begin{aligned} u_1 &= -(\bar{r} - r)x_1 + \bar{r}\bar{k}x_2^2 - rkx_1^2 \\ &\quad + \bar{a}_1 x_2 y_2/(1+\bar{b}_1 x_2) - a_1 x_1 y_1/(1+b_1 x_1) \\ &\quad + v_1, \\ u_2 &= -(d_1 - \bar{d}_1)y_1 - \bar{a}_1 x_2 y_2/(1+\bar{b}_1 x_2) \\ &\quad + a_1 x_1 y_1/(1+b_1 x_1) + \bar{a}_2 y_2 z_2/(1+\bar{b}_2 y_2) \\ &\quad - a_2 y_1 z_1/(1+b_2 y_1) + v_2, \\ u_3 &= -(d_2 - \bar{d}_2)z_1 - \bar{a}_2 y_2 z_2/(1+\bar{b}_2 y_2) \\ &\quad + a_2 y_1 z_1/(1+b_2 y_1) + v_3. \end{aligned} \quad (6)$$

Then, the error system (5) becomes

$$\begin{cases} \dot{e}_1 = \bar{r}e_1 + v_1, \\ \dot{e}_2 = -\bar{d}_1 e_2 + v_2, \\ \dot{e}_3 = -\bar{d}_2 e_3 + v_3. \end{cases} \quad (7)$$

The error system (7) is linear and the convergent solution can be found under appropriate control input v_1, v_2 and v_3 as functions of e_1, e_2 and e_3. As long as the solutions of system (7) converge to zero as time t goes to infinity, the synchronization of two multi-trophic ecological chaotic systems will be achieved. There are many possible choices for v_1, v_2 and v_3 control functions. They are proposed as follows:

$$\begin{aligned} v_1 &= -(\bar{r} + k_1)e_1, \\ v_2 &= (\bar{d}_1 - k_2)e_2, \\ v_3 &= (\bar{d}_2 - k_3)e_3, \end{aligned} \quad (8)$$

where k_1, $k_2$and k_3 are positive control gains. Now, the error system (7) becomes

$$\begin{cases} \dot{e}_1 = -k_1 e_1, \\ \dot{e}_2 = -k_2 e_2, \\ \dot{e}_3 = -k_3 e_3. \end{cases} \quad (9)$$

The error system (9) can be rewritten as $\dot{e} = Ae$, where

$$A = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 0 \\ 0 & 0 & -k_3 \end{bmatrix}. \quad (10)$$

The eigenvalues of Aare all negative, thereforee$_1$, e_2 and $e_3$converge to zero as time t goes to infinity. This implies that the synchronization of two multi-trophic ecological chaotic systemswith uncertain parameters is achieved by means of the active controllers in Eqs. (6) and (8).

IV. NUMERICAL SIMULATIONS

In this section, numerical simulations are demonstrated to validate the theoretical analysis. The designed synchronization signals based on active control technique are added to the response chaotic multi-trophic ecological system. The initial and parameter values given in Section 2 are considered for chaotic trajectories. Thecontrol gains are taken as$k_1 = k_2 = k_3 = 1$. Fig. 7 and Fig. 8 show the time history of synchronized multi-trophic ecological system having Hastings–Powell parameter values in the drive system (3) and Stone–He parameter values in the response system (4) with the active controllers are activated at t = 200s andt = 300s, respectively. In addition, the synchronization errors are demonstrated in Fig. 9.

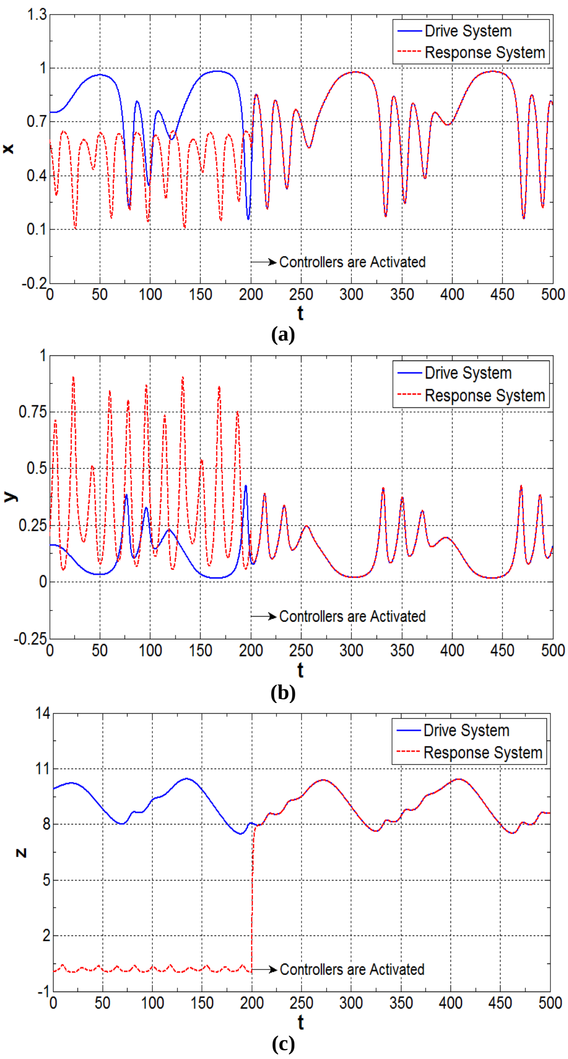


Fig. 7.Synchronized multi-trophic ecological systemshaving Hastings–Powell parameter values in the drive system and Stone–He parameter values in the response system with the active controllers are activated at t = 200s in (a) x, (b) y, (c) z time series.

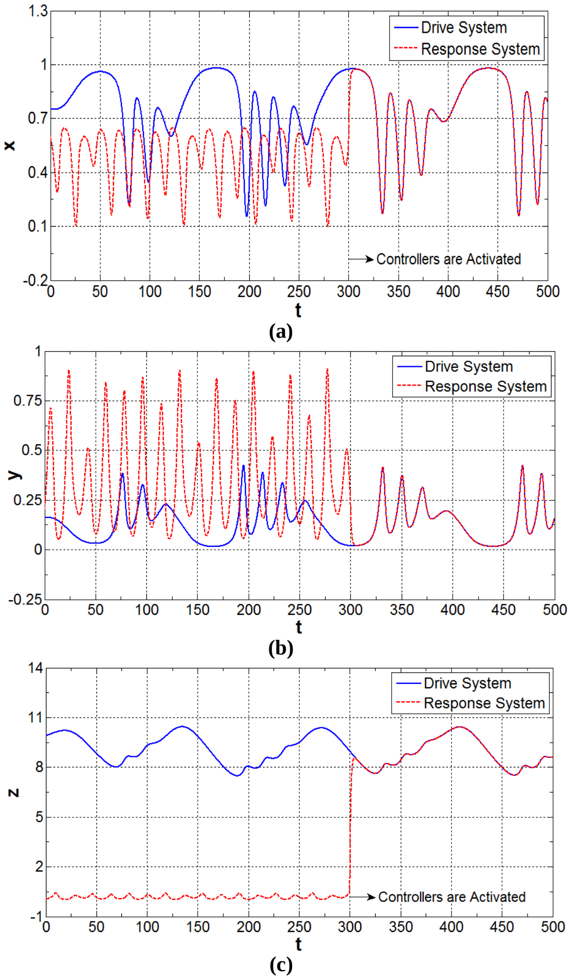


Fig. 8.Synchronized multi-trophic ecological systemshaving Hastings–Powell parameter values in the drive system and Stone–He parameter values in the response system with the active controllers are activated at t = 300s in (a) x, (b) y, (c) z time series.

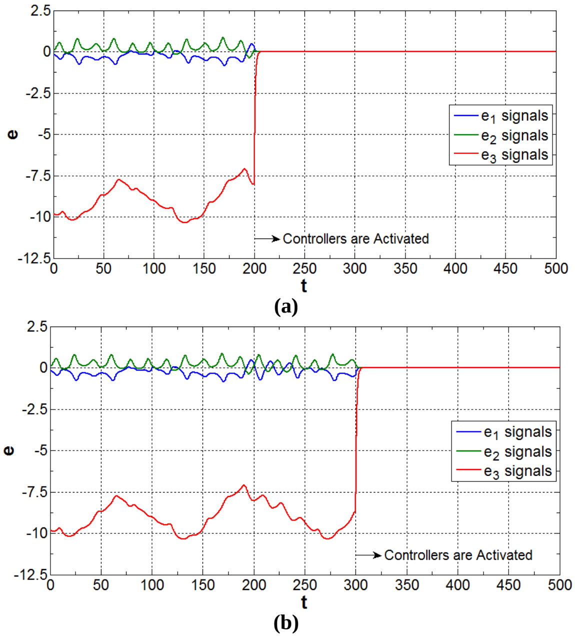


Fig. 9.Synchronization errors of multi-trophic ecological systemshaving Hastings–Powell parameter values in the drive system and Stone–He parameter values in the response system with the active controllers are activated at (a)t = 200s,and (b) t = 300s.

Fig. 10 and Fig. 11 show the time history of synchronized multi-trophic ecological system having Stone–He parameter values in the drive system (3) and Hastings–Powell parameter values in the response system (4) with the active controllers are activated at t = 200s andt = 300s, respectively. In addition, the synchronization errors are demonstrated in Fig. 12.

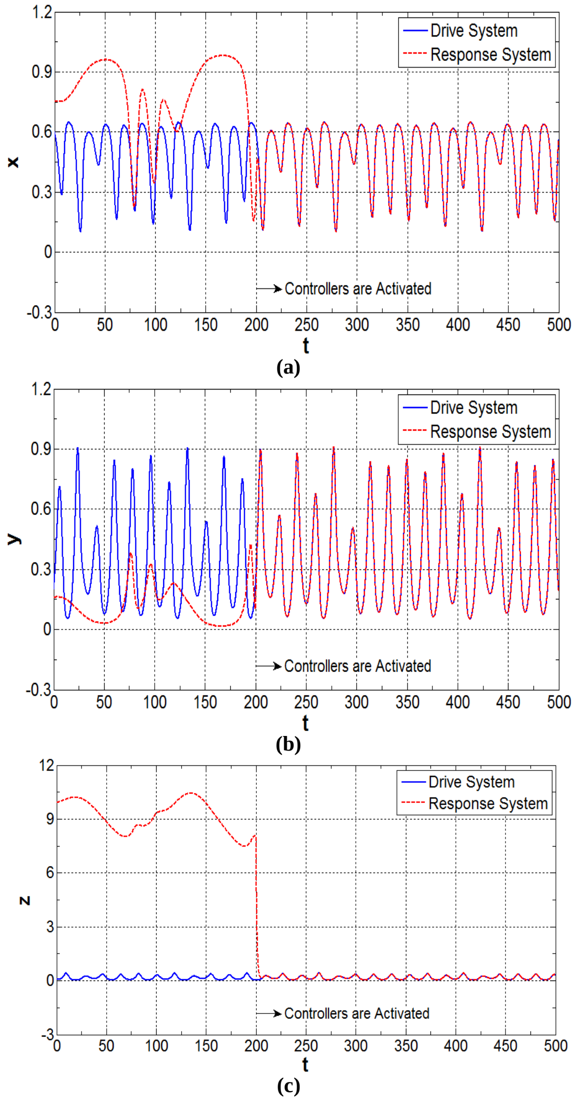
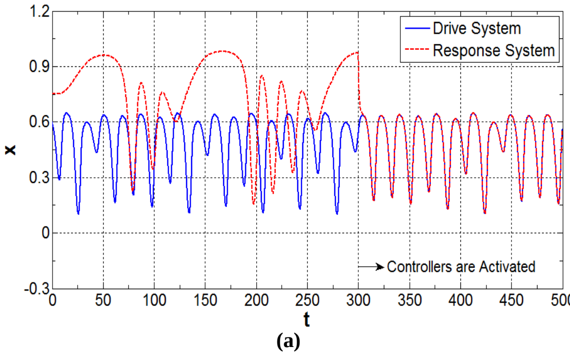
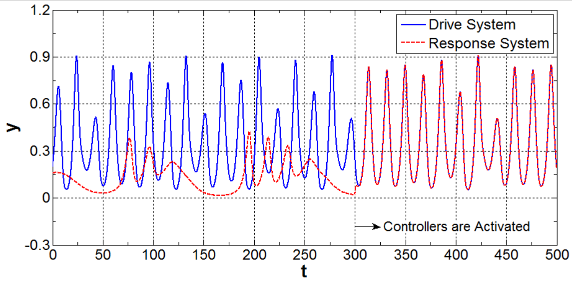
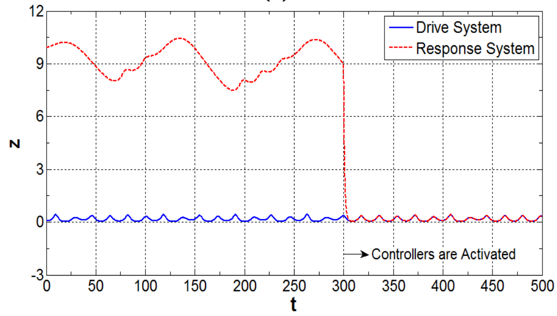


Fig. 10.Synchronized multi-trophic ecological systemshaving Stone–He parameter values in the drive system and Hastings–Powell parameter values in the response system with the active controllers are activated at t = 200s in (a) x, (b) y, (c) z time series.



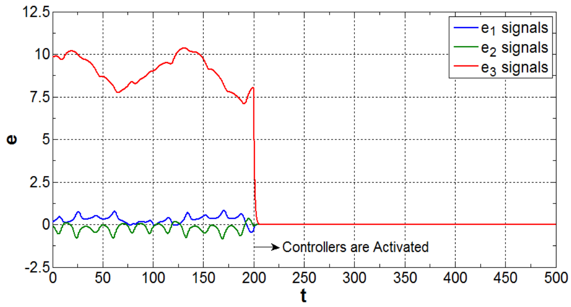


(b)

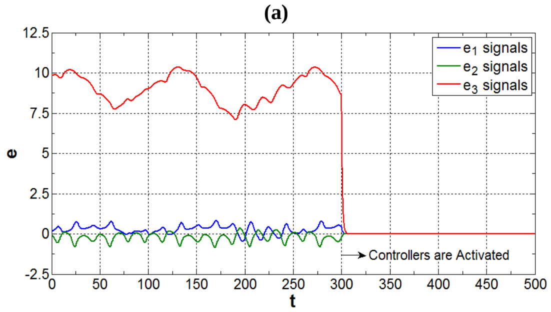


(c)

Fig. 11.Synchronized multi-trophic ecological systemshaving Stone–He parameter values in the drive system and Hastings–Powell parameter values in the response system with the active controllers are activated at t = 300s in (a) x, (b) y, (c) z time series.



(a)



(b)

Fig. 12.Synchronization errors of multi-trophic ecological systemshaving Stone–He parameter values in the drive system and Hastings–Powell parameter values in the response system with the active controllers are activated at (a)t = 200s,and (b) t = 300s.

As shown in Figs.7–12, the synchronization ofchaotic multi-tropic ecological systemsis achieved and the synchronization errorsasymptotically converge to zero after the active controllers are activated.Therefore, the theoretical analysis is confirmed.Simulations results also show that the designed active controllers realize the chaos synchronizationin an appropriate time period.

CONCLUSIONS

In this study, thesynchronization of chaos in a multi-trophic ecological system model with uncertain parameters is investigated. Using the active theory, three controllers are added to the response system to apply the chaos synchronization. Based on the active control theory, the conditions of asymptotic stability at the zero synchronization errorpoint are obtained theoretically. Some numerical simulations are demonstrated and theyverify the theoretical analysis. Furthermore, simulation results show the effectiveness of proposed synchronizationtechnique.

ACKNOWLEDGMENTS

This study was supported by Research Fund of Sakarya University.

REFERENCES

[1] E.N. Lorenz,“Deterministic nonperiodic flow,” J. of Atmospheric Sciences, vol. 20, no. 2, pp. 130–141, 1963.

[2] W.J. Zhou, Z.P. Wang, M.W. Wu, W.H. Zheng, and J.F. Weng,“Dynamics analysis and circuit implementation of a new three-dimensional chaotic system,” Optik,vol. 126, no. 7–8,pp. 765–768, 2015.

I. Bodale andV.A. Oancea,“Chaos control for Willamowski–Rossler model of chemical reactions,” Chaos, Solitons & Fractals, vol. 78, pp. 1–9, 2015.

[3] J.N. Wang and W.H. Jiang, “Bifurcation and chaos of a delayed predator-prey model with dormancy of predators,” Nonlinear Dynamics, vol. 69, no. 4, pp. 1541–1558, 2012.

[4] El-Gohary, “Chaos and optimal control of equilibrium states of tumor system with drug,” Chaos, Solitons & Fractals, vol. 41, no. 1,pp. 425–435, 2009.

[5] Q. Gao and J.H. Ma, “Chaos and Hopf bifurcation of a finance system,” Nonlinear Dynamics, vol. 58, issue 1–2, pp. 209–216, 2009.

[6] L.M. Pecora and T.L. Carroll, “Synchronization in chaotic systems,” Physical Review Letters, vol. 64, no. 8, pp. 821–824, 1990.

[7] E.W. Bai and K.E. Lonngren, “Synchronization of two Lorenz systems using active control,” Chaos, Solitons & Fractals, vol. 8, no. 1, pp. 51–58, 1997.

[8] H.N. Agiza and M.T. Yassen, “Synchronization of Rossler and Chen chaotic dynamical systems using active control,” Physics Letters A, vol. 278, no. 4, pp. 191–197, 2001.

[9] G.H. Li,“An active control synchronization for two modified Chua circuits,” Chinese Physics, vol. 14, no. 3, pp. 472–475, 2005.

[10] Goksu, U.E. Kocamaz, and Y. Uyaroglu,“Synchronization and control of chaos in supply chain management,” Computers & Industrial Engineering, vol. 86, pp. 107–115, 2015.

[11] U.E. Kocamaz, A. Goksu, H.Taskin, and Y. Uyaroglu, “Synchronization of chaos in nonlinear finance system by means of sliding mode and passive control methods: A comparative study,” Information Technology and Control, vol. 44, no. 2, pp. 172–181, 2015.

[12] S. Vaidyanathan, “Adaptive control and synchronization of Rössler prototype-4 system,” Int. J. of Advanced Information Technology, vol. 1, no.5, pp. 11–21, 2011.

[13] Hastings and T. Powell, “Chaos in a three-species food chain,” Ecology, vol. 72, no. 3, pp. 896–903, 1991.

[14] L. Stone and D. He, “Chaotic oscillations and cycles in multi-trophic ecological systems,” J. of Theoretical Biology, vol. 248, no. 2, pp. 382–390, 2007.

★★★