SUPPLY CHAIN NETWORK DESIGN USING CONSTRAINT PROGRAMMING

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Abstract- Supply chain network offers an efficient service design by meeting demands of customers with minimum cost for the retailers. The term of the design in supply chain management contains the selection of suppliers including manufacturers, distribution center and carriers for the transportation of products from producers to stores. This paper provides two mathematical modelling alternatives for designing supply chain networks: integer programming (IP) and constraint programming (CP). After presenting the general environment of the supply chains, two approaches are adapted to the problem in a hypothetical case. The results of the numerical studies show that CP is a considerable alternative approach on designing supply chain networks.

Index Terms- Constraint Programming, Integer Programming, Supply Chain Network Design,

I. INTRODUCTION

Supply chain strategy is a critical factor for a company's position in the competition. Competitive advantages in the supply chain could provide companies unique competencies which are hard to imitate in the industry. Sitek and Wikarek [1] describe the main issue to tackle in supply chain design as the minimization of costs while creating a constant stream of goods, services and information. Optimization efforts dealing with this problem could include a mix of different layers and agents in the supply chain.

Modeling the decision of choosing an optimal path for the flow of goods and information entails an optimization of combinatorial nature generally. Furthermore, this issue is tackled with mathematical programming approach in the literature.

The growth of constraints along with growing complexity is highlighted in Sitek and Wikarek [1] and is addressed via the Constraint Programming (CP) approach. The authors believe the interdepency of the constraints in sustainable supply chain management (SCM) could be tackled with constraint programming approach, leading to better solutions.The production and delivery of a variety of products at low cost, high quality and profit is only possible with effective design and management of supply chain networks [2].

This study will examine the incorporation of different products and their suppliers. Furthermore, the integer programming (IP) model describing the supply chain network (SCN) will be modeled and examined with constraint programming approach and CP search strategies.

TheSCN comprise levels representing the different physical locations and actors involved in a supply chain. A realistic model, which incorporates more than two levels, are more complex and involves a greater number of decisions. When a problem of this nature is addressed with optimization tools, it can be asserted that they should represent a supply chain strategy.

The outline of this is structured as follows: Section 2 presents the related literature and the contribution of the paper.Section 3 presents the problem statement, integer programming model formulation and constraint programming model with search strategies for the problem.Section 4 provides a numerical study with alternative scenarios and compares the solution qualities of the approaches. Finally, Section 5 discuss the conclusionsand future research direction of the study.

II. LITERATURE REVIEW

A. Constraint Programming

"CP is built upon constraints and constraint solving" [3].The problem is defined by using the constraints on it by the user. This approach leads to a more intuitive and natural description of the problem [4]. Another aspect of the CP models is that they represent the problem types and that they can be reused with different parameters.

Furthermore, CP models address problems of combinatorial nature. These problems can be NP-hard and generally require great amount of computer time. CP approaches can reduce the solution space with the description of the problem, leading to shorter solution times.

Jussien and Barichard [4]state: "A constraint is a logical relation among variables with different domains" and they go on to claim that constraints reduces the domains dictating the defined relations.
Constraints have two roles; namely, to ensure the domain values satisfy the constraint and to prune the

ones that do not satisfy it [4]. The user has a much smaller search space when all the relations are implemented and irrelevant values are pruned.

While the CP approach has been a computer science topic until recently, it has started to be exploited for real life problems. Jussien and Barichard [4]point out three contemporary trends in CP;

- Utilizing the CP approach in new types of problems
- Extending the power of the constraints and designing new global constraints for problems of different types
- Making the CP available as a tool to tackle all engineering problems.

B. Supply Chain Management

SCM,which has become very popular in recent years, is the organization of the flow of supplies and demands within and across the companies in a network[5-10].Decision makers of SCM's face various problem types in different levels while establishing a supply chain. Strategic level problems mainly include marketing strategies,technology andstrategic options for the future, tactical level problems include supplier selection and evaluation for the competitionand operational level include deployment, routing and scheduling problems for the current situation[11]. The supply chain design problem (SCDP)covers strategic and tactical level goals[12]. In general, the SCDP aims to coordinately combine suppliers, distribution centers (DC) and transportationchains to satisfy the demands of customers and retailers in an optimal way. Several extensions on the SCDP have been published in recent years [11, 13-18].

C. Supply Chain Management Literature Incorporating CP

As stated in Jussien and Barichard [4], the utilization of CP algorithms in different problems is increasing. SCM is an area where the CP models are incorporated in optimization problems, heuristics and also hybrid problems where they are used in conjoint with traditional mathematical programming. Sitek and Wikarek [1] claim they believe in the effectiveness of the declarative CLP environment in describing a solid framework for all the aspects of the problem. They go on to use CP for reducing their search space and solve a mathematical programming model in the reduced domain. This kind of approach deals with the elimination of infeasible solutions first and finding the optimal solution is dealt with later. It is clear that the CP model can be utilized for reducing the domain of the decision variables and this is further improved with search strategies.

Li and Jiang [19]on the other hand address the uncertainty in the SCN by solving the safety stock problem with CP algorithm. They claim to have been transformed the safety stock problem into a project scheduling problem Li and Jiang [19] and since the CP is efficient for modeling scheduling problems, there are specific constraints for scheduling in OPL, they have solved the problem with constraint programming.

Jussien and Barichard [4]point out to the fact that a set of standard search procedures exist in CP and that new techniques are being developed. Apart from developing new functions suitable for real world problems, the exploration of search space is another domain of constraint programming research. A hybrid approach is adopted in Li and Jiang [19]; they use genetic algorithm for a faster exploration of the search space. The authors claim that they obtained better and more efficient solutions. The possibility of using metaheuristics and machine learning methods for the search space exploration highlights the fact that the CP is a computer programming approach. Mak, et al. [20], use CP in conjoint with simulated annealing for solving a supplier selection problem. The authors claim that the companies are seeking supply chain strategies for increasing their service level while decreasing costs [20]. They implement a CP model for solving this complex problem entailing several objectives and interdependent constraints.
This paper contributes to the advancement of the literature dealing with designing SCN using CP.

III. PROBLEM STATEMENT AND FORMULATION

This paper deals with the SCDP based on different products that are sold in different stores retrieved from various suppliers in the network. This model is settled for a general SCNcontaining of four different levels(Fig. 1). The first level hosts stores*(c)* that sell various products*(i)*.The second levelconsists ofDCs*(d)* that stock and transport products between manufacturers and stores. The third levelhosts carriers*(t)*that distribute products amongst manufacturers, DCs and stores. The manufacturers *(s)*with possible different capabilities is the last level.

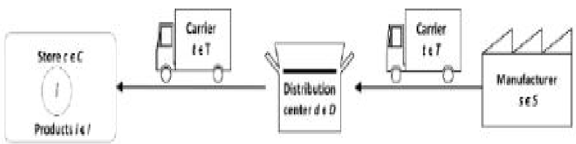

Fig. 1: A general SCNP settlement

This problem is firstly modeled by using integer programming approach with the objective function of maximizing the total utility by satisfying the aim of meeting the right product to the right store in the network. The total utility of the SCDP is calculated using the total income and total processing, transportation cost via DC from manufacturer and store fromthe DC.There are some assumptions made in order to concentrate on themain interest of the study:

- Lost sales are allowed.
- No cost associated with store or retailer management.
- One planning period is considered.
- Demands are not divisible.
- The unit sizes of different products are the equal.

Set & Indices

$i \in I$	Set of products
$s \in S$	Set of manufacturers
$d \in D$	Set of DCs
$c \in C$	Set of stores
$t \in T$	Set of carriers

Parameters

e_i	Selling price of product *i*
q_i^s	Unit production cost of manufacturer *s* for product *i*
h^t	Unit transportation cost of carrier *t*
k^d	Unit storage cost of DC*d*
y^s	Production capacity of manufacturer *s*
z^d	Storage capacity of DC*d*
v^t	Transportation capacity of carrier *t*
a_c^d	Distance between store *c* and DC*d*
b^{ds}	Distance between DC*d* and manufacturer *s*
de_{ci}	Demand of store *c* for product *i*

Decision Variables

X_{ci}^{tds}	1: if store *c* for product *i* by using carrier *t* and DC*d* from manufacturer *s* uses this path; 0: otherwise.

IP MODEL

$$\max\left(\begin{array}{l}\sum_c\sum_i\sum_t\sum_d\sum_s X_{ci}^{tds} de_{ci} e_i - \\ \sum_c\sum_i\sum_t\sum_d\sum_s X_{ci}^{tds} de_{ci}\left(q_i^s + k^d + b^{ds}h^t + a_c^d h^t\right)\end{array}\right) \quad (1)$$

The objective function (1) aim to maximize the total utility of the SCNby considering the total income and costs.

$$\sum_c\sum_i\sum_t\sum_d de_{ci} X_{ci}^{tds} \le y^s \qquad \forall s \in S \quad (2)$$

$$\sum_c\sum_i\sum_t\sum_s de_{ci} X_{ci}^{tds} \le z^d \qquad \forall d \in D \quad (3)$$

$$\sum_c\sum_i\sum_d\sum_s de_{ci} X_{ci}^{tds} \le v^t \qquad \forall t \in T \quad (4)$$

$$\sum_t\sum_d\sum_s X_{ci}^{tds} \le 1 \quad \forall c \in C, i \in I \quad (5)$$

$$X_{ci}^{tds} \in [0,1] \quad \forall c \in C, i \in I, t \in T, d \in D, s \in S \quad (6)$$

Equations (2), (3) and (4) are the capacity constraints for manufacturers, carriers and DCs, respectively. Constraint (5) defines the demands are not divisible. Finally, constraint (6) defines the variable domain.

CP MODEL

While the only decision variable is a boolean variable with multiple indices in the IP Model, the constraint programming model entails more decision variables with a singleboolean variable. The SCDP is a combinatorial problemwhich involves the selection of an optimal combination of supply network components. Accordingly, different options are allocated to different stages of the problem. The allocation is handled in the CP model via integer decision variables which could take the values of the index of the set the choice is made.

The model entails the linking of the manufacturer, carrier and DC to the stores where there is a fixed demand for the products. The most obvious assumption of the model is that there would be no unmet demand. This further leads to our revenue calculation where all the demand is calculated as an actual sale.

Since CP requires a verbal description of the problem, we have chosen to allocate the demand values to the suppliers, carriers and DCs. The parameter$de_{ci}$is a matrix with indices*c* and *i*. Therefore, the allocation decision variablesα, β andδ need to have the same indices, this way, the value of the decision variables would be the indices of the target stage.

Three decision variables unique to CP Model are as follows:

$\alpha_{ci} \in S$	The value is equal to the index of the supplier (S) which would satisfy the demand for store c and product *i*.
$\beta_{ci} \in D$	The value is equal to the index of the DCs (D) which would satisfy the demand for store c and product *i*.
$\delta_{ci} \in T$	The value is equal to the index of the carrier (T) which would satisfy the demand for store c and product *i*.

These decision variables are used as index values in the constraints. The Boolean decision variableY^{tds}takes the value of 1 if carrier *t* and DC*d* from manufacturer *s* uses this path; 0 otherwise.This variable makes sure the amount of demand where the allocation made becomes effective.

Furthermore, while the expressions such as revenue and costs are stated as constraints, in the CP model, these are expressed as decision expressions in OPL.

$$max\left(\begin{array}{l}\sum_c\sum_i Y^{\alpha_{ci}\beta_{ci}\delta_{ci}} de_{ci} e_i - \\ \sum_c\sum_i Y^{\alpha_{ci}\beta_{ci}\delta_{ci}} de_{ci}\left(\begin{array}{l} q_i^{\alpha_{ci}} + k^{\beta_{ci}} \\ + b^{\alpha_{ci}\beta_{ci}} h^{\delta_{ci}} + a_c^{\alpha_{ci}} h^{\delta_{ci}}\end{array}\right)\end{array}\right) \quad (7)$$

In the objective function (7), the gross profit is calculated by subtracting all manufacturing and transportation costs from the revenue.

$$\sum_{c}\sum_{i} Y^{s\beta_{ci}\delta_{ci}} de_{ci} \le y^{s} \qquad \forall s \in S \tag{8}$$

$$\sum_{c}\sum_{i} Y^{\alpha_{ci}d\delta_{ci}} de_{ci} \le z^{d} \qquad \forall d \in D \tag{9}$$

$$\sum_{c}\sum_{i} Y^{\alpha_{ci}\beta_{ci}t} de_{ci} \le v^{t} \qquad \forall t \in T \tag{10}$$

$$Y^{tds} \in [0,1] \qquad \forall t \in T, d \in D, s \in S \tag{11}$$

$$\alpha_{ci} \in S, \beta_{ci} \in D, \delta_{ci} \in T \qquad \forall c \in C, i \in I \tag{12}$$

Equations (8), (9) and (10) are the capacity constraints for manufacturers, carriers and DCs, respectively. Finally, constraint (11) and (12) defines the variable domain.

IV. COMPUTATIONAL STUDY

D. Problem Data

The test data setsusedto compare performance of the approaches are given Table 1 which shows number of products, manufactures, stores, DCs and carriers, respectively.

Table I: Data sets

	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set10
i	1	1	2	1	3	4	2	1	3	4
s	1	2	2	1	3	3	2	5	3	5
c	2	2	2	5	2	2	3	4	3	4
d	2	2	2	3	2	2	3	2	3	2
t	2	2	2	3	2	2	3	3	3	3

The data related to Set 10 are detailed in Table II, III, IV and V.In this set, there are four different products, five manufacturers with different production capabilities, four stores, twoDCs and three carrier alternatives. Table II presents the data related to products, including demand and sale prices. Production costs and maximum production capacity of manufacturers are given in Table III. Table IV presents the data related to carriers. DC data consist of distances, capacities and storage costs are shown in Table V.

Table II: Data related to products

i	de_{1i}	de_{2i}	de_{3i}	de_{4i}	e_i
1	100	100	0	0	30
2	200	0	200	0	15
3	300	0	300	500	20
4	0	50	0	0	40

Table III: Data related to manufacturers

s	q_1^s	q_2^s	q_3^s	q_4^s	y^s
1	10	6	4	13	500
2	20	10	4	20	500
3	8	6	4	13	500
4	20	3	8	20	500
5	20	20	4	20	500

Table IV: Data related to carriers

t	v^t	h^t
1	1000	0.005
2	1000	0.002
3	1000	0.008

Table V: Data related to DCs

d	b^{d1}	b^{d2}	b^{d3}	b^{d4}	b^{d5}	a_1^d	a_2^d	a_3^d	a_4^d	z^d	k^d
1	476	66	1012	626	182	173	503	314	158	1000	0.2
2	222	358	782	514	136	287	332	257	449	1000	0.5

E. Comparison of IP and CP Models

The related models have been coded by using IBM ILOG Optimization Studio 12.6, run in a workstation which has Intel Xeon 2.5 GHz CPU with 8 GB of RAM and solved by using CPLEX and CP solvers. The “Restart” search type and “Depth First” method used for solving the CP model. The results presented in Table VI show that the CP model able to solve sample sets optimally as long as the total number of constraints are less than 1818.

After this number the IP model continues to solve sample sets optimally. But the CP model found feasible solution. However, the difference between the results of CP and IP model is less than 5 percent of the largest sample set. Furthermore, both approaches solve the problem quickly since this is a small problem in size

In this study, “Restart” method was expected to perform better, however it is close to “Depth First” method. Although the search speed of Restart is higher, it does not produce the optimal solution for this model.

Table VI:Performance comparison between IP and CP Models

		Set1	Set2	Set3	Set4	Set5	Set6	Set7	Set8	Set9	Set10
IP	Total Profit	3689,4	2771,4	5189,8	6478,6	9721,2	10934,8	6494	5814,3	15453	23.675,5
	Number of constraints	12	13	15	17	18	20	19	19	23	31
	Number of decision variables	14	22	38	51	78	102	114	126	249	486
	Solution Time(s)	0	0,02	0	0,02	0,01	0,03	0,01	0	0,01	0,05
CP	Total Profit	3689,4	2771,4	5189,8	6478,6	9714,4	10928	6494	5814,3	15374,6	21989,9
	Number of constraints	380	425	849	1090	1366	1818	1463	1101	2334	4397
	Number of decision variables	10	14	20	24	30	36	36	42	54	78
	Solution Time(s)	0,09	0,98	1,01	1,01	100,01	100,03	100,01	100,02	100,02	100,01

CONCLUSION

This work uses integer programing and constraint programing (CP) approach to design supply chain networks. The performance of the alternative modelling approaches is compared under different problem configurations. The results show that CP model should be configured with different value, variable ordering and search strategies since the problem domain contains so many constraints. One of the major contributions of this study is to exploit the effect of the CP strategies on solution quality of the problem. Future research directions may include integration of IP and CP modeling along with heuristic approaches for possible improvements.

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