CONTROLLING THE CHAOTIC MOTION OF FLOW IN A THERMAL CONVECTION LOOP WITH SLIDING MODE CONTROL

[1]HAKAN KIZMAZ, [2]UGUR ERKİN KOCAMAZ, [3]YILMAZ UYAROGLU

[1]Department of Electrical & Electronics Engineering, Faculty of Engineering and Architecture,Siirt University, Siirt, Turkey
[2]Department of Computer Technologies, Vocational School of Karacabey, Uludağ University, Karacabey, Bursa, Turkey
[3]Department of Electrical & Electronics Engineering, Faculty of Engineering, Sakarya University, Serdivan, Sakarya, turkey
E-mail: [1]hakankizmaz@siirt.edu.tr, [2]ugurkocamaz@gmail.com, [3]uyaroglu@sakarya.edu.tr

Abstract: This study deals with controlling the chaotic motion of flow in a thermal convection loop by using a sliding mode control signal. Time series, two-dimensional phase plots and three-dimensional phase plane of the chaotic flow in the thermal convection are presented graphically from its differential equations. Based on the sliding mode control theory, a Lyapunov function is assigned for ensuring the global asymptotic stability of the error system. Routh–Hurwitz criterions are also used for determining the signs of sliding mode parameters. Simulation results are demonstrated to verify the correctness of the theoretical results. They also show that the chaotic motion of flow in a thermal convection loops effectively controlled owing to a sliding mode controller.

Keywords: Chaos, Chaos Control, One State Controller, Sliding Mode Control, Thermal Convection Loop.

INTRODUCTION

Chaotic systems can be defined as a kind of deterministic nonlinear systems which include unpredictable and irregular dynamics. They behave alike stochastic systems. Their trajectories sensitively depend on initial conditions. Chaos is a very interesting phenomenon in a variety of fields including physics, chemistry, ecology, biology, and finance [1–5]. Many engineering applications also involve chaotic behaviours [6–8].

Chaotic control is also an important topic and paying attentions in physics, mathematics and engineering. Ott, Grebogi and Yorke published a paper which presents a chaos control strategy in 1990 [9]. After their pioneer work, chaos has been shown to be controllable. The method in [9] is named as OGY method. Many other control methods have been published for chaos control such as linear feedback, nonlinear feedback, time-delay feedback, active, adaptive, sliding mode, optimal, passive, backstepping, neural, and fuzzy control. Among them, sliding mode control is known as a robust control technique against of uncertainties in a control system. It also demonstrates effective controlling nonlinear systems like chaotic systems. Many researchers have focused on sliding mode control of nonlinear or chaotic systems. The sliding mode control method has been successfully applied for the control of Lorenz [10], Chua [11], Rössler [12], Duffing–Holmes [13], and many other chaotic systems [14].

A thermal convection loop is generally constructed from a pipe bent into a torus and standing in the vertical plane. Chaotic behaviour can be observed in such system when the heating rates exceed a certain threshold value. Chaos is an unwanted phenomenon for this situation. It causes some temperature vibrations which can increase drag in flow systems and exceed safe operational conditions in thermal systems. Therefore, eliminating the chaotic oscillations and making the flow approximately stable have significant importance. In some papers, the chaos in the thermal convection loop is successfully controlled by making small adjustments to the heating rate in response to events detected inside the loop [15–20]. Feedback [15], active [16, 17], optimal and adaptive [18], nonlinear active feedback [19], linear and nonlinear feedback [20] control methods have been used. In this study, the sliding mode control method is investigated.

II. THERMAL CONVECTION LOOP

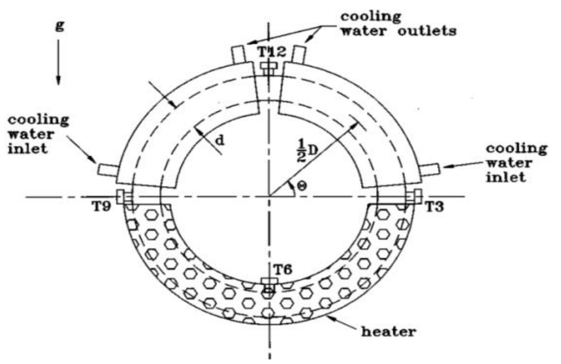


Fig. 1. The thermal convection loop [18].

The schematic description of thermal convection loop is shown in Fig. 1 where g, d, D, θ, and T_w represent the gravitational acceleration, the diameter of the pipe, the diameter of the torus, the angular location of a point on the torus, and the wall temperature of the pipe, respectively. It is observed that when the temperature differences of horizontal and vertical positions are measured as time function, heating and cooling temperature values can cause fluid movement in the system.

The thermal convection loop is defined by a set of three autonomous differential equations [18]:

$$\left.\begin{aligned} \dot{x} &= p(y-x), \\ \dot{y} &= -xz-y, \\ \dot{z} &= xy-z-r, \end{aligned}\right\} \qquad (1)$$

where the state variables x, y, and z represent the cross-sectional averaged speed, proportional to the fluid's temperature differences between positions 3 and 9 o'clock, and positions 12 and 6 o'clock around the loop, respectively. p is the loop's Prandtl number and r is the loop's Rayleigh number. The thermal convection loop displays chaotic behaviour when the parameter values are taken as p = 4 and r = 16 with the initial conditions x_0 = 1, y_0 = 1, and z_0 = –9.9 [18].

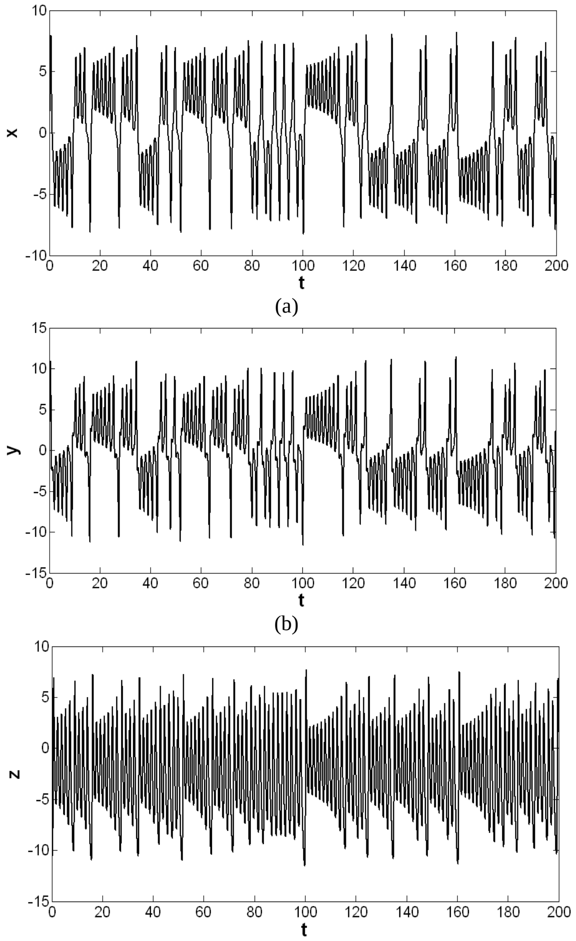


Fig.2. The time series of the chaotic thermal convection loop for (a) *x* signals, (b) *y* signals, (c) *z* signals.

The time series of the chaotic thermal convection loop are shown in Fig. 2, the phase plots are shown in Fig. 3 and the 3D phase plane is shown in Fig. 4.

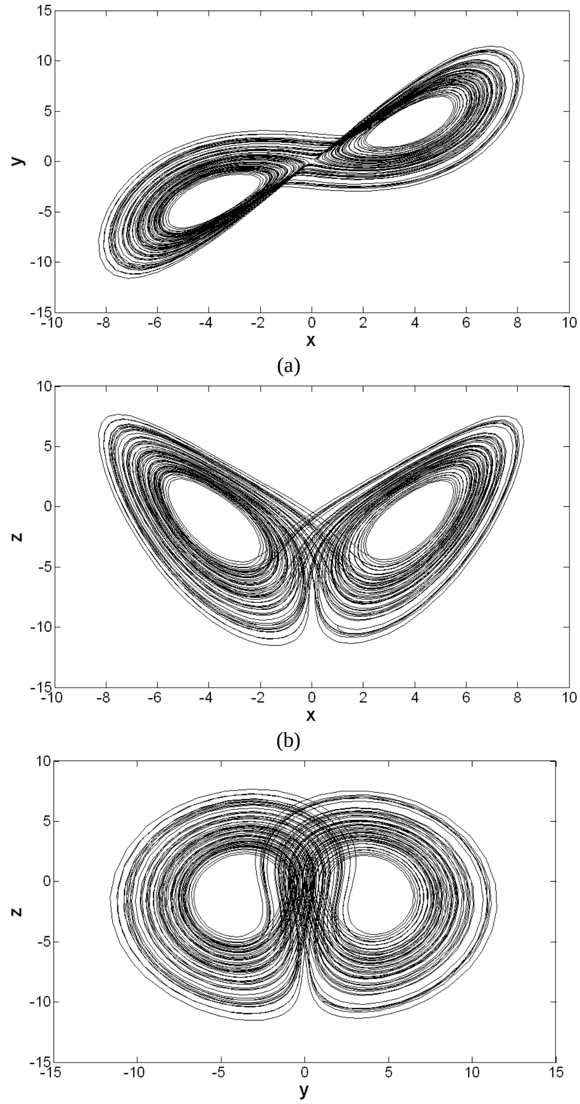


Fig. 3. The phase plots of the chaotic thermal convection loop for (a) *x–y* phase plot, (b) *x–z* phase plot, (c) *y–z* phase plot.

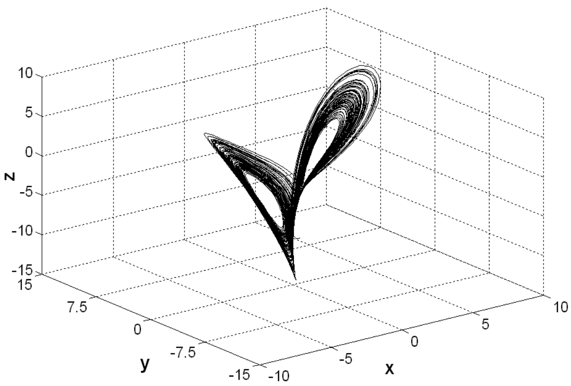


Fig. 4. 3D phase plane of the chaotic thermal convection loop.

The equilibrium point of chaotic thermal convection loop system can be found by assuming $\dot{x}$=0, $\dot{y}$=0, $\dot{z}$=0, and solving the following equation:

$$\left.\begin{aligned} p(y-x) &= 0, \\ -xz-y &= 0, \\ xy-z-r &= 0. \end{aligned}\right\} \qquad (2)$$

Thus, system (1) has three equilibrium points: $E_1(0,0,-r)$, $E_2(\sqrt{r-1}, \sqrt{r-1}, -1)$, and $E_3(-\sqrt{r-1}, -\sqrt{r-1}, -1)$.

III. CONTROL WITH SLIDING MODE CONTROL

In order to control the chaotic motions of flow in the thermal convection loop to its equilibrium points, a sliding mode control signal u is added to the system (1). Hence, the system including the control signal becomes

$$\left.\begin{aligned} \dot{x} &= p(y-x), \\ \dot{y} &= -xz-y+u, \\ \dot{z} &= xy-z-r. \end{aligned}\right\} \tag{3}$$

An equilibrium point can be presented as (x_d, y_d, z_d), then the trajectory error states are determined as $e_1 = x - x_d$, $e_2 = y - y_d$ and $e_3 = z - z_d$. Thus, the state variables become $x = e_1 + x_d$, $y = e_2 + y_d$ and $z = e_3 + z_d$. The error state dynamic equations of system (3) can be expressed by

$$\left.\begin{aligned} \dot{e}_1 &= p(e_2 - e_1 + y_d - x_d), \\ \dot{e}_2 &= -z_d e_1 - e_2 - x_d e_3 - e_1 e_3 - y_d - x_d z_d + u, \\ \dot{e}_3 &= y_d e_1 + x_d e_2 - e_3 + e_1 e_2 - z_d + x_d y_d - r. \end{aligned}\right\} \tag{4}$$

As a result of $y_d - x_d = 0$, $-y_d - x_d z_d = 0$ and $-z_d - x_d y_d - r = 0$, the error system (4) can be simplified as

$$\left.\begin{aligned} \dot{e}_1 &= p(e_2 - e_1), \\ \dot{e}_2 &= -z_d e_1 - e_2 - x_d e_3 - e_1 e_3 + u, \\ \dot{e}_3 &= y_d e_1 + x_d e_2 - e_3 + e_1 e_2. \end{aligned}\right\} \tag{5}$$

The system (5) clearly shows that if e_2 is zero, then $\dot{e}_1=-pe_1$. Thus, if time goes to infinite, e_1 will converge to zero. Hence, the required sliding surface may be chosen as

$$s = e_2 + k_1 e_3, \tag{6}$$

where k_1 is a real constant parameter. When $s\dot{s}$ is negative for all cases of $s \neq 0$, sliding mode reaching condition is obtained. The sliding mode control signal can be constructed as follows:

$$\left.\begin{aligned} u = {} & z_d e_1 + e_2 + x_d e_3 + e_1 e_3 \\ & - k_1 (y_d e_1 + x_d e_2 - e_3 + e_1 e_2) \\ & - k_2 s - k_3 \operatorname{sign}(s), \end{aligned}\right\} \tag{7}$$

where k_2 and k_3 are real constant parameters. The sign(s) is signum function where

$$\operatorname{sign}(s) = \begin{cases} 1, & \text{if } s > 0, \\ 0, & \text{if } s = 0, \\ -1, & \text{if } s < 0. \end{cases} \tag{8}$$

The proposed control signal (7) assures that system (5) is onto the sliding surface $s = 0$. The derivative of sliding surface s is

$$\dot{s} = -k_2 s - k_3 \operatorname{sign}(s). \tag{9}$$

A Lyapunov function is taken as $V = 0.5s^2$. Its derivative with respect to time becomes

$$\dot{V} = s\dot{s} = -k_2 s^2 - k_3 (s) \operatorname{sign}(s). \tag{10}$$

Therefore, if $k_2 \geq 0$ and $k_3 \geq 0$, then $-k_2 s^2 \leq 0$ and $-k_3(s)\operatorname{sign}(s) = -k_3|s| \leq 0$. Since $V \geq 0$ and $\dot{V} \leq 0$, according to Lyapunov stability theory, the designed sliding surface s would globally converge to the zero error point.

Substituting Eq. (7) into system (5) gives the following error dynamics

$$\left.\begin{aligned} \dot{e}_1 &= p(e_2 - e_1), \\ \dot{e}_2 &= -k_1 (y_d e_1 + x_d e_2 - e_3 + e_1 e_2) \\ &\quad - k_2 (e_2 + k_1 e_3) - k_3 \operatorname{sign}(e_2 + k_1 e_3), \\ \dot{e}_3 &= y_d e_1 + x_d e_2 - e_3 + e_1 e_2. \end{aligned}\right\} \tag{11}$$

The Jacobian matrix of system (11) is:

$$J = \begin{bmatrix} -p & p & 0 \\ -k_1 y_d & -k_1 x_d - k_2 & k_1 - k_1 k_2 \\ y_d & x_d & -1 \end{bmatrix}. \tag{12}$$

For $E_1(0, 0, -r)$, the characteristic equation of matrix (12) is calculated as follows:

$$(-p-\lambda)(-k_2-\lambda)(-1-\lambda) = 0. \tag{13}$$

From Eq. (13), it is easy to obtain that $\lambda_1 = -1$. Since $p > 0$ and $k_2 \geq 0$, the other eigenvalues are also negative.

For $E_2(\sqrt{r-1}, \sqrt{r-1}, -1)$, the characteristic equation of matrix (12) is calculated as follows:

$$\left.\begin{aligned} & \lambda^3 + (p + k_1\sqrt{r-1} + k_2 + 1)\lambda^2 \\ & + (p + k_2 + pk_2 + 2pk_1\sqrt{r-1} + k_1 k_2 \sqrt{r-1})\lambda \\ & + (pk_2 + 2pk_1 k_2 \sqrt{r-1}) = 0. \end{aligned}\right\} \tag{14}$$

For $E_3(-\sqrt{r-1}, -\sqrt{r-1}, -1)$, the characteristic equation of matrix (12) is calculated as follows:

$$\left.\begin{aligned} & \lambda^3 + (p - k_1\sqrt{r-1} + k_2 + 1)\lambda^2 \\ & + (p + k_2 + pk_2 - 2pk_1\sqrt{r-1} - k_1 k_2 \sqrt{r-1})\lambda \\ & + (pk_2 - 2pk_1 k_2 \sqrt{r-1}) = 0. \end{aligned}\right\} \tag{15}$$

According to the Routh–Hurwitz stability criterion, for a third-order system

$$\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0, \tag{16}$$

all the eigenvalues are negative and the system will be stable if the coefficients satisfy

$$\left.\begin{array}{l} c_1 > 0, c_2 > 0, c_3 > 0, \\ c_1 c_2 > c_3. \end{array}\right\} \qquad (17)$$

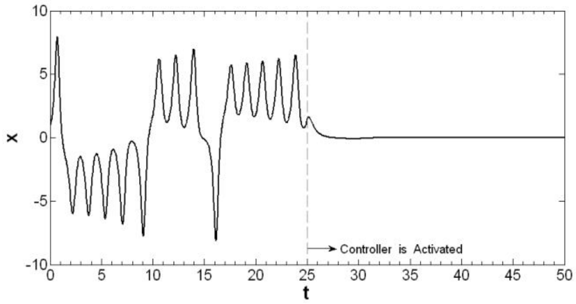
Since $p>0$, $r>0$, and $k_2 \geq 0$, the Routh–Hurwitz stability criterions are provided with $k_1 \geq 0$ for E_2 and $k_1 \leq 0$ for E_3. Therefore, the global asymptotical stability of system (5) towards its equilibrium points is sustained by employing the sliding mode control method with the control function in Eq. (7) and the k_1 conditions.

Hence, the sliding mode control of chaotic thermal convection loop system (3) is completed.

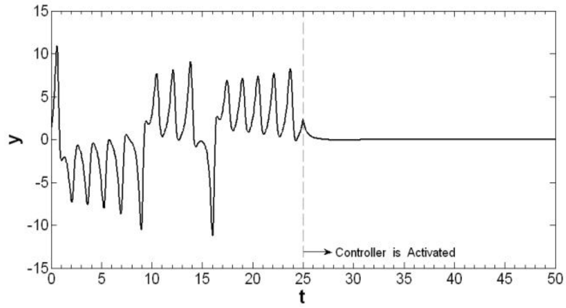
IV. NUMERIC SIMULATIONS

The fourth order Dormand–Prince method with variable time-step is used in all the numerical simulations. The previously mentioned parameter set and initial conditions are considered to ensure the chaotic behaviour of motion flow in a thermal convection loop. The sliding mode controller are activated at t = 25 in all simulations. The parameter k_1 is taken as $k_1 = 1$ for the equilibrium points $E_1(0, 0, -r)$ and $E_2(\sqrt{r-1}, \sqrt{r-1}, -1)$, and $k_1 = -1$ for the equilibrium point $E_3(-\sqrt{r-1}, -\sqrt{r-1}, -1)$. The other gains of the sliding mode controller are considered as $k_2 = 1$ and $k_3 = 0.1$. Simulation results for the control of chaotic thermal convection loop system (3) towards equilibrium points E_1, E_2, and E_3 by means of a sliding mode control signal are shown in Fig. 5, Fig. 6, and Fig. 7, respectively.

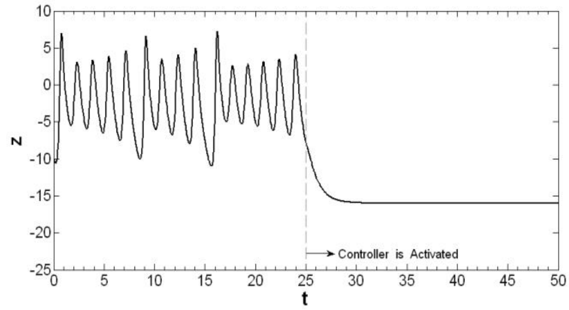
The Figs. 5–7 show that the chaotic motion of flow in a thermal convection loop is controlled to its equilibrium points, after the sliding mode controller is activated. So, the simulation results confirm the theoretical analysis. Numerical simulations also show the effectiveness of the proposed one state sliding mode controller. When it is activated at t = 25 with the unit gains, the control is completely observed at $t \geq 32$ for the equilibrium point $E_1(0, 0, -r)$, and at $t \geq 29$ for the equilibrium points $E_2(\sqrt{r-1}, \sqrt{r-1}, -1)$ and $E_3(-\sqrt{r-1}, -\sqrt{r-1}, -1)$. The signal z plays significant role in the control performances. Hence, simulation results have validated the effectiveness of the proposed controller in the control of chaotic thermal convection loop system.



(a)

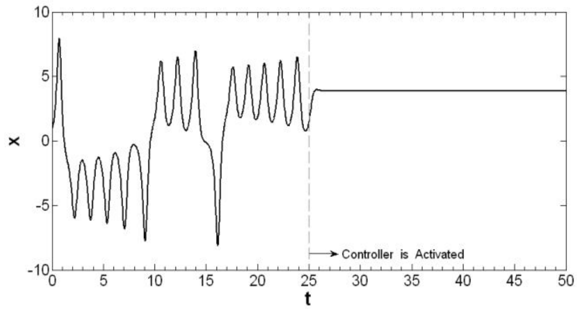


(b)

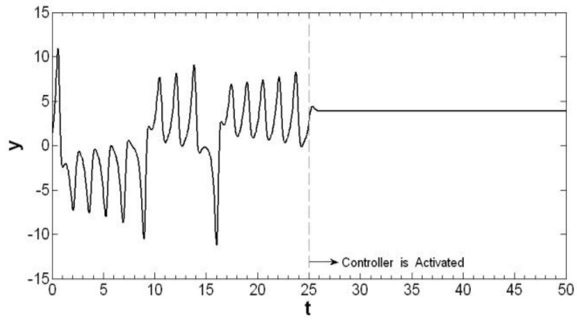


(c)

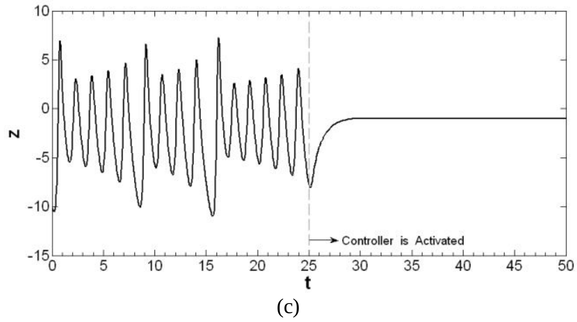
Fig. 5. The time series of the chaotic thermal convection loop for the equilibrium point E_1 when the sliding mode controller is activated at t = 25 for (a) x signals, (b) y signals, (c) z signals.



(a)



(b)



(c)

Fig. 6. The time series of the chaotic thermal convection loop for the equilibrium point E_2 when the sliding mode controller is activated at t = 25 for (a) x signals, (b) y signals, (c) z signals.

CONCLUSIONS

In this study, chaos control of flow in a thermal convection loop is investigated. Based on the Lyapunov stability theory and the Routh–Hurwitz criterions, an appropriate sliding surface is designed for suppressing the chaos in a three-dimensional thermal convection loop. It is theoretically proofed that a single state sliding mode control signal can be sufficient for the control. Then, numerical simulations have been carried out to verify the effectiveness of proposed control strategy. They have shown that the chaotic thermal convection system stabilizes towards its $E(0, 0, -r)$ equilibrium point in 7-time period and the other two equilibrium points in 4-time period by means of a sliding mode control signal having unit gains.

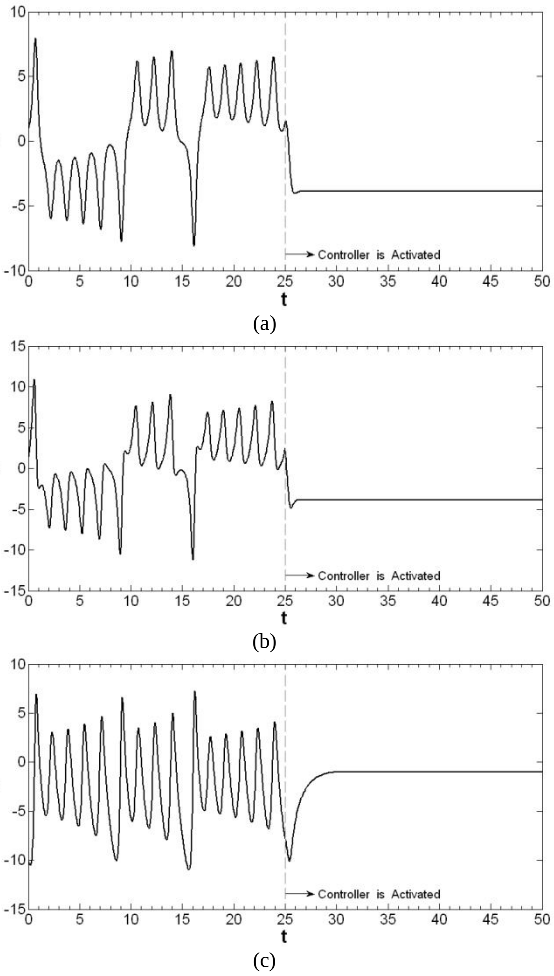


Fig. 7. The time series of the chaotic thermal convection loop for the equilibrium point E_3 when the sliding mode controller is activated at t = 25 for (a) x signals, (b) y signals, (c) z signals.

REFERENCES

[1] Zhou WJ, Wang ZP, Wu MW, Zheng WH, Weng JF. Dynamics analysis and circuit implementation of a new three-dimensional chaotic system. Optik 2015; 126(7–8): 765–768.

[2] Bodale I, Oancea VA. Chaos control for Willamowski-Rossler model of chemical reactions. Chaos, Solitons & Fractals2015; 78: 1–9.

[3] Ali N, Chakravarty S. Stability analysis of a food chain model consisting of two competitive preys and one predator. Nonlinear Dynamics2015; 82(3): 1303–1316.

[4] El-Gohary A. Chaos and optimal control of equilibrium states of tumor system with drug. Chaos, Solitons & Fractals 2009; 41(1): 425–435.

[5] Zhang XD, Liu XD, Zheng Y, LiuC. Chaotic dynamic behavior analysisand control for a financial risk system. Chinese Physics B2013; 22(3): 030509.

[6] Cuomo KM, Oppenheim AV. Circuit implementation of synchronized chaos with applications to communications. Physical Review Letters1993; 71(1): 65–68.

[7] Akgul A, Calgan H, Koyuncu I, Pehlivan I, Istanbullu A.Chaos-based engineering applications with a 3D chaotic system without equilibrium points. Nonlinear Dynamics 2016; 84(2): 481–495.

[8] Liu HJ, Kadir A, Li YL. Asymmetric color pathological image encryption schemebased on complex hyper chaotic system. Optik 2016; 127(15): 5812–5819.

[9] Ott E, GrebogiC, Yorke JA. Controlling chaos. Physical Review Letters 1990; 64(11): 1196–1199.

[10] Yau HT, Yan JJ. Design of sliding mode controller for Lorenz chaotic system with nonlinear input.Chaos, Solitons & Fractals 2004; 19: 891–898.

[11] Jang MJ, Chen CL, Chen CK. Sliding mode control of chaos in the cubic Chua's circuit system. International Journal of Bifurcation and Chaos 2002; 12(6): 1437–1449.

[12] Chang JF, Hung ML, Yang YS, Liao TL, Yan JJ. Controlling chaos of the family of Rössler systems using sliding mode control. Chaos, Solitons & Fractals 2008; 37: 609–622.

[13] Yau HT, Chen CK, Chen CL. Sliding mode control of chaotic systems with uncertainties. International Journal of Bifurcation and Chaos 2000; 10(5): 1139–1147.

[14] KonishiK, Hirai M, KokameH. Sliding mode control for a class of chaotic systems. Physics Letters A, 1998; 245: 511–517.

[15] Singer J, Wang YZ, Bau HH. Controlling a chaotic system. Physical Review Letters 1991; 66(9): 1123–1125.

[16] Singer J, Bau HH. Active control of convection. Physics of Fluids A–Fluid Dynamics 1991; 3(12): 2859–2865.

[17] Wang Y, Singer J, Bau HH.Controlling chaos in a thermal convection loop. Journal of Fluid Mechanics1992; 237: 479–498.

[18] Yuen PK, Bau HH.Optimal and adaptive control of chaotic convection – Theory and experiments. Physics of Fluids1999; 11(6): 1435–1448.

[19] Yang L, Zhang YS,Xiao H, Wan SM.The nonlinear active control of chaos in thermal convection loop. International Conference on Energy Conversion and Application, Wuhan, China,June 17–20, 2001, pp. 453–456.

[20] Yang L, Wang SM, Zhang YS, Jiang XQ. Linear and nonlinear feedback control of chaos in a thermal convection loop. Heat Transfer– Asian Research2002; 31(6): 430–437.

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