RESONANCE FREQUENCY VARIATIONS OF METALLIC TIBETAN SINGING BOWL WITH TEMPERATURE

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Abstract- Metallic signing bowls are used in sound therapy and in Buddhist prayers are widely studied for their resonance frequency modes [13,2]. Studies have also been carried out to understand the resonance frequency variations effects of the bowl partly filled with water and the resulted changes in the resonance modes were studied without considering the coupling effects of the metal-fluid interfaces and fluid-fluid interfaces[13].Also Nonlinear effects and the temperature variation effects are not considered. In addition Finite elements based models (FEM) have been developed to analyze the higher order modes [2,15] of the bowl. However all these FE models do not consider the torsional mode of vibration and the nonlinear effects due to temperature and fluids even though they are significant in the metallic singing bowls. Also very limited work has been done to understand the temperature variations of the singing bowl resonance frequencies. First part of this paper describes the variation of the frequency modes of both empty and partly filled bowls respect to temperature variations. A significant variation of resonance frequency modes have been observed with the experimented temperatures from 5^0C to 40^0C, which is the commonly used temperature range for the singing bowls. Also comparison of the simulation results with experimental data are discussed. The second part of this paper discusses a nonlinear FE analysis of the metallic singing bowls with metal-fluid and fluid-fluid coupling layers elements. The simulated resonance modes and the measured resonance modes results are compared in this paper.

Index Terms- Metallic Singing Bowls, Structural-Fluid, Acoustic Coupling, Nonlinear Finite Element modelling.

I. INTRODUCTION

Acoustic resonators have been used in Buddhist practices [16] and ancient healing rituals. Out of many kinds of ancient metallic singing bowls, there are three types which are widely used in Buddhist practices (Fig 1.a, Fig 1.b and Fig1.c). They are the Tibetan version (Fig 1.a), the Chinese version (Fig 1.b) and the Indian version (Fig 1.c). Exact manufacturing history of metallic singing bowls still not known [13]. During this study only a version of metallic Tibetan singing bowl is selected, that is been commonly used in sound therapy (Fig 1.a). Exact martials constituents and composition of these Bowls are not yet know, however metallic analysis have found that they are comprising of multitude of metals including copper, tin, zinc, iron, silver, gold and nickel (Bronze alloys) [13,2].Out of many excitation methods, rubbing outer ring of the bowls excitation method is used. For the analytical modelling of the resonance modes I have used the French et al [14] work as previous researchers have done [13] and in addition considered the torsional vibrating modes, Temperature effects and the nonlinear fluid effects [14]. Moreover finite elements modelling (FEM) has been carried out by the researchers[13,2] to understand complex vibrational modes did not consider the structural-fluid and fluid-fluid acoustic coupling effects and also only the nonlinear effects of the excitation is considered with respect to nonlinear nature of the problem. This study also carried out a nonlinear FEM based modelling, in addition to the nonlinear effects of the excitation method, I have considered the structural-fluid and fluid-fluid acoustic coupling, temperature effects and nonlinear effect of the fluid on the resonance modes of the singing bowl.

Fig 1.a Tibetan Singing Bowl Fig 1.b Chinese Singing Bowl Fig 1.c Indian Singing Bowl

II. ANALYTICAL TEMPERATURE MODEL OF THE SINGING BOWL

It is assumed that the bowl is vibrating both horizontal plane and vertical plane as shown in the Fig2.0. Previous researches have observed that the oscillation modes of the bowl resembles that of the wine glasses [13] without taking the geometrical variations between the wine glass and the signing bowl. Therefore the conservation of energy based multimodal linear analytical model by French et.al. [14,3,4,6,9,10] of the wine glass resonances can be modified and applied to analyze the bowl. Assumed that the vertical modes are n and horizontal modes are m. Where, X,Y,Z are the orthogonal coordinate system, (0,0,0) coordinate is at the center of the base of the bowl, H is the height, R_L is the initial top of the bowl radius, R_s is the bowl base radius, $\boldsymbol{\theta}$ is the azimuth angle and $\boldsymbol{\Delta}$ is the amplitude of the oscillation, F(z) is the bowl side edge geometry profile transformation displacement with respect to the height z which is monotonically increasing one, fluid filled height is taken as h and thickness of the bowl is taken as a [14]. The edge effect curvature is not considered. The oscillating frequency is taken as $\boldsymbol{\omega}$, then the following equation 2 can be taken as the generalized equation of radial vibration for the bowl and the frequency variations due to the partly filed bowl is analyzed by using the generalized equation 3. These two equations are the generalized form of French et. al.[14] equations for the fundamental form. Then these two equations are used to formulate the vibration modes frequencies. Equations 2 and 3 are used to develop the temperature dependent equations and hence obtain the frequency variation analytical models.

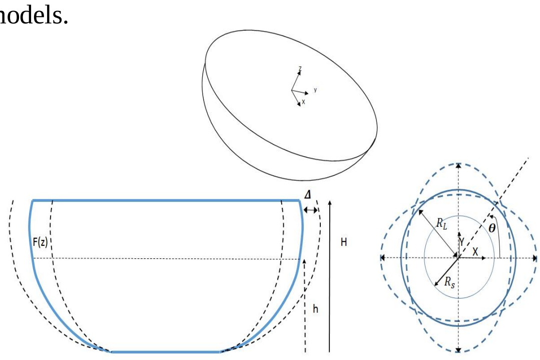

Fig 2.0 Dimensions and the geometry (isometric, side and top views)of the Bowl

The radial direction displacement r(t) is given by,

$$r(t) = \Delta\cos(\omega t)\cos(n\theta)\,F(z) - - - - - (1)$$

Then total energy (E) of the system of small volume is given by assuming an isothermal system.

E= Kinetic Energy + Potential energy

$$E = A\left(\frac{d\Delta}{dt}\right)^2 + B\Delta^2 - - - - - - - - - - - (1.1)$$

Where A and B are taken as constants and from the Law of conservation energy substituting the radial displacement equation in to the energy equation 1.1 the vibration mode frequencies can be obtained by the following equation 1.2. Where A and B are constant for a given mode of vibration at a given temperature.

$$\omega^2 = {}^{B}/_{A} - - - - - - - - (1.2)$$

Then the modes of vibrating frequency for the empty bowl given by equation 2,

$$f_{n,m} = \frac{\omega_{n,m}}{2\pi} = \frac{1}{12\pi}\left(\frac{3Y}{p_b}\right)^{1/2}\frac{a}{R_L{}^2}\left[\frac{\left((n^2-1)+\left({}^{mR_L}/_{H}\right)^4\right)}{\left(1+{}^{1}/_{n^2}\right)}\right]^{1/2} - (2)$$

Where Y is the Young's modulus, p_b is the density of the bowl. For the wine glass model of French it is approximated that the $F(z) = \left({}^{z}/_{Z}\right)^{3/2}$ as the bowl edge profile different form that of the wine glass and also the relative deformation is more with the metallic bowls F(z) is approximated for the bowl as $F(z) = \left({}^{z}/_{Z}\right)^{k}$ where the constant k is found by plotting the profile F(z) against z and then by using least square regression approximation. At z=0 deformation is taken as 0. K is approximated as 5/4. From the French et al equations [14] it can be easily shown that the A is given by equation 2.1,

$$A = \left(\frac{\pi}{2}(p_b a R_L)\left(1+{}^{1}/_{n^2}\right) + \alpha\frac{\pi}{8}p_l R^2\left[\frac{\int_0^h F(z)^2 dz}{\int_0^H F(z)^2 dz)}\right]\right)\Delta^2 - (2.1)$$

Can be simplified as equation 2.2,

$$A = \left(\frac{\pi}{2}(p_b a R_L)\left(1+{}^{1}/_{n^2}\right) + \alpha\frac{\pi}{8}p_l(R_L)^2\frac{16}{41}\left(\frac{h}{H}\right)^{41/16}\right)\Delta^2 - (2.2)$$

Where α *is a constant* dependent on the boundary conditions between the bowl surface and fluid for a perfect coupling $\alpha = 1$ and p_l is the density of the fluid. From French et al [14] B is given by the equation 2.3,

$$B = \frac{\pi}{24}\left(\frac{Ya^3}{R_L{}^3}\right)\left((n^2-1)+\left({}^{mR_L}/_{H}\right)^4\right)\Delta^2 - (2.3)$$

Then the resonance modes frequencies are obtained from the following approximated equation 3.

$$f_{n,m} = \frac{1}{12\pi}\sqrt{\frac{B}{A}} = \frac{1}{12\pi}\left(\frac{\left(\frac{3Ya^3}{R_L{}^2}\right)\left((n^2-1)+\left({}^{mR_L}/_{H}\right)^4\right)}{\left((p_b a)\left(1+{}^{1}/_{n^2}\right)+\alpha p_l R_L\frac{16}{164}\left(\frac{h}{H}\right)^{41/16}\right)}\right)^{1/2} - (3)$$

Since Bronze alloys tends to reduce its young's modulus with the increase of the temperature [8].We assumed the Young's modulus variation with temperature Y(T) is given by the equation 3.1 and its value is reducing with temperature. At temperature from 5^0C to 40^0C we can assume that the reduction of the young's modulus is linear with first order reduction coefficient α_Y.

$$Y(T) = Y_5(1-\alpha_Y T) - - - - - - (3.1)$$

Where Y_5 is the Young's modulus at 5^0C and T is the Temperature difference in C^0 from 5^0C.

Since it is a metallic bowl we assumed material is linear homogeneous and isotropic so that R_L *and* H are increasing linearly with the temperature and are given by the following equation 3.2 and 3.3.

$$R_L(T) = R_{L_5}(1+\alpha_{R_L}T) - - - - - - (3.2)$$

$$H(T) = H_5(1+\alpha_H T) - - - - - - - (3.3)$$

Where R_{L_5} and H_5 are respective values at 5^0C and α_{R_L} and α_H are linear thermal expansion coefficients. The densities p_b *and* p_l [7,8] are decreasing with the temperature according to the following equation 3.4 and 3.5, respectively, where α_{p_b} is the thermal reduction coefficient.

$$p_b(T) = p_{b_5}(1-\alpha_{p_b}T) - - - - - - - (3.4)$$

$$p_l(T) = 999.85308 + 6.32693x10^{-2}T - 8.523829x10^{-3}T^2 + 6.943248x10^{-5}T^3 - 3.821216x10^{-7}T^4 - -(3.5)$$

Also since the fluid thermal expansion is more than the metallic expansion the h(T) also increasing and we assume that to be linear and given by the equation 3.6, where α_h is the thermal expansion coefficient.

$h(T) = h(1 + \alpha_h T) - - - - - - - - - - (3.6)$

Thickness (a) variation not considered as it is smaller compare to the other dimensions.

By using the 3 and 3.1 to 3.6 equations the temperature dependent resonance frequency modes can be found by using the equation 4.

$$f_{n,m} = \frac{1}{12\pi}\left(\frac{\left(\frac{3Y(T)a^3}{R_L(T)^2}\right)\left((n^2-1)+\left({}^{mR_L(T)}/_{H(T)}\right)^4\right)}{\left((p_b(T)a)\left(1+{}^1/_{n^2}\right)+ap_l(T)R_L(T)\frac{16}{164}\left(\frac{h(T)}{H(T)}\right)^{41/16}\right)}\right)^{1/2} - (4)$$

Assuming ${}^{R_L(T)}/_{H(T)}$ comparable and geometrical volumes of the bowl and fluid and $R_L(T)$ *and* $h(T)$ *increasing and* $Y(T)$ *reducing* it can be seen that the temperature variation of $f_{n,m}$ is negative that means as the temperature increases the resonance modes frequencies are reducing of the singing bowl under both empty and the partly filled conditions. Young's modulus Y is estimated by using the resonance frequency modes at n=2 and m=0 at a given temperature. Also other dimensions are also measured at different temperatures and temperature coefficients were estimated.

III. IMPROVED NONLINEAR FINITE ELEMENT (FE) BASED MODEL

However analytical solution discussed in the previous chapter does not taken into the account the nonlinear effects. Nonlinear finite element methods based solutions have been put forward by researchers [13,2]. A nonlinear FE method introduced by the Octávio Inácio et al [2] does not taken into the account of the nonlinear effects due to temperature, acoustic pressure and structural-acoustic coupling at the fluid-bowl and fluid-fluid contacts regions but only consider the nonlinear effects of the force at contact points. However the nonlinear effects due to the force contact points have limited importance when considering the resonance modes of the bowl as the resonance modes depend on the nonlinear Eigen value equation, that depend on the physical properties of the context. Therefore I have improved nonlinear model given by Octávio Inácio et al[2] by introducing the temperature effects, nonlinear materials effects and structural acoustic coupling elements for fluid-solid and fluid-fluid contact regions. I have used the structural acoustic coupling elements introduced by Giulio Maier et al [21] assuming elastic structure. The finite element solution of structural acoustic coupling problem was formulated by Carlsson [22].

The sketch of the structural acoustic coupling problem is shown in the Fig 2.0. The problem consist of two mediums structure and fluid namely and their respective boundary conditions are marked in the Figure 2.0. Mathematical formulation is done with the following force equation in elastic solid continuum medium and the boundary conditions. Strains of fluid and structure and fluid boundary condition where assumed to be equal and the pressure and the normal stress at fluid-structure boundary are also assumed to be equal.

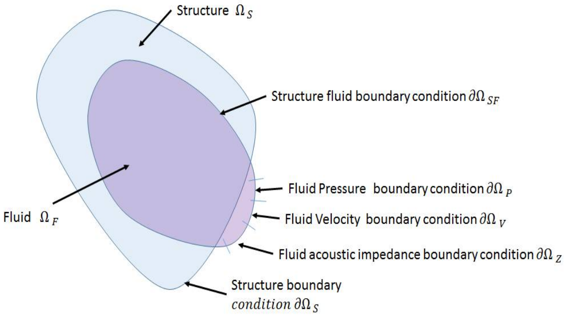

Fig 2.0 Diagram of Structural Acoustic Coupling Problem

Structural medium $\tilde{\nabla}^T\sigma_S + b_s = \rho_s \frac{d^2U_S}{dt^2} \quad in\,\Omega_S$ and $\partial\Omega_S$ and initial conditions ----(5)

Where, σ_S is the stresses, b_s is the body force density, ρ_s is the density and U_S is the displacement.

Fluid mediums including the air and the water the KZK equation is used [19,1,5,11,12]

$$\frac{\partial^2\Psi}{\partial^2 t} - c_0^2\nabla^2\Psi = \frac{\partial}{\partial t}\left(b\nabla^2\Psi + (\nabla\Psi)^2 + \frac{{}^D/_E}{2c_0^2}\left(\frac{\partial\Psi}{\partial t}\right)^2\right). \quad in\,\Omega_F$$

$$b = \frac{1}{\rho_0}\left(\frac{4}{3}\mu + \mu_B\right) + \frac{k}{\rho_0}\left(\frac{1}{C_v} - \frac{1}{C_p}\right),$$

$$p = p_0 + \rho_0\left(\frac{\partial\Psi}{\partial t} - \frac{(\nabla\Psi)^2}{2} + \frac{1}{2c_0^2}\left(\frac{\partial\Psi}{\partial t}\right)^2\right) - \left(\frac{4}{3}\mu + \mu_B\right)\nabla^2\Psi$$

Where Ψ is the acoustic velocity potential, D and E are the coefficients of the first and the second terms of the Tailor series expansion of the equation relating material pressure (P) to it's density (ρ). K is the thermal conduction, μ, μ_B viscosity and the bulk viscosity, Where, p_0 is the initial pressure, c_0 is the acoustic velocity and ρ_0 is the ambient density and C_p is specific heat capacity constant pressure and C_v is specific heat capacity constant volume.

and $\partial\Omega_P, \partial\Omega_V, \partial\Omega_Z$ initial conditions. –(6)

Coupling boundary $U_S|_n = U_F|_n$ displacement at the boundary, $\sigma_S|_n = -P$ is the stress at the boundary,--(7)

Coupling boundary $U_{Water}|_n = U_{Air}|_n$ displacement at the boundary, $P_{Water} = -P_{Air}$ is the pressure at the boundary. The finite element solution is given by the Equation 8.1 [1,18]. In the solution of the model problem second order approximation of nonlinear longitudinal wave equation is employed. A set of super nodes which facilitates direct coupling with the structure and the cavity mass is employed.

$$\begin{bmatrix} M_s & 0 \\ 0 & M_F \end{bmatrix}\begin{bmatrix} \Delta\ddot{U} \\ \Delta\ddot{\psi} \end{bmatrix} + \begin{bmatrix} R_s & L \\ -\rho c^2 L^T & R_F \end{bmatrix}\begin{bmatrix} \Delta\dot{U} \\ \Delta\dot{\psi} \end{bmatrix} + \begin{bmatrix} K_S & -H \\ 0 & K_F \end{bmatrix}\begin{bmatrix} \Delta U \\ \Delta\Psi \end{bmatrix} = \begin{bmatrix} Res_s \\ Res_F \end{bmatrix} - -(8.1)$$

Where,
M_s *is the assembled structural mass matrix*,
$\rho c^2 L^T, L$ are the assembled fluid structure coupling mass matrix, where ρ *is the density an c is the velocity of sound*
K_S is the assembled structural stiffness matrix,
K_F *is the assembled fluid equivalent stiffness matrix*
$-H$ is the assembled coupling stiffness matrix, F_b is the assembled body force matrix
F_q *is the assembled fluid flow force matrix*
R_s is the assembled solid damping matrix
R_F *is the assembled fluid damping matrix*
Res_s *Residuals of solid and* Res_F
Residuals of Fluid (Air and Water),U and P are nodal displacement and pressure matrices

To obtain the acoustic resonance modes and frequencies reduced Eigen equation 8.2 [17,18] is used.

$$\begin{bmatrix} M_s & 0 \\ 0 & M_F \end{bmatrix}\begin{bmatrix}\Delta\ddot{U}\\ \Delta\ddot{\psi}\end{bmatrix} + \begin{bmatrix} R_s & L \\ -\rho c^2 L^T & R_F\end{bmatrix}\begin{bmatrix}\Delta\dot{U}\\ \Delta\dot{\psi}\end{bmatrix} + \begin{bmatrix}K_S & -H\\ 0 & K_F\end{bmatrix}\begin{bmatrix}\Delta U\\ \Delta\Psi\end{bmatrix} = \begin{bmatrix}Res_s\\ Res_F\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \quad --(8.2)$$

First step was the formulation of the M_s and K_S by assembling all the structural acoustic elements. The second step was the formulation of M_F and K_F by assembling the acoustic fluid elements including the water and air. The third step was the formulation of R_s and R_F by assembling the structural and fluid acoustic elements including water and Air. Third fourth was the formulation of H by using the coupling between the solid-water, solid-Air and water-Air boundary conditions. The boundaries solid –water and the solid air assumed to be coupled element with damper elements [20]. The water Air boundary is assumed to be having coupled elements by considering the surface tension [ref]. At the solid fluid boundary it is assumed that the degree of freedom (DOF) of solid, water and air acoustic elements are normal to the fluid-fluid boundary it is assumed that the normal pressure is balanced with the surface tension. The final step of the formulation was the assembling the Eigen equation 8.2 for obtaining the resonance modes of acoustic vibrations. The initial conditions assumed to be zero at the boundaries to obtain the Eigen solution. The elements of the matrices of the equation 8 are functions with respect to temperature. The nonlinear Eigen value equation matrices element values are estimated at temperature 5^0C to 40^0C and solutions are found for frequencies from 10Hz to 10kHz. I have used a popular open source nonlinear finite element solver for the implementation. The temperature dependent nonlinearity of the dimension of the bowl and the material properties are estimated from the equations 3.1 through 3.6. The sound velocity in air C_{Air} is also assumed to be nonlinear with temperature and equation 9.1 is used for the estimation of the velocity considering the 343m/s at 293 K^0 (20 C^0).

$$C_{Air} = \left(\frac{\Upsilon RT}{M}\right)^{1/2} ----(9.1)$$

Where $\Upsilon = \frac{c_p}{c_v}$.

In fluid the velocity is estimated by using the pressure and the density nonlinear relationships and by using the following equation 9.2 assuming isentropic process.

$$(C_{water})^2 = \frac{\partial P}{\partial \rho} ----(9.2)$$

Three dimensional acoustic tetrahedral elements are selected for meshing and nonlinear FE solver is used for the simulation the meshing diagrams are shown in following Figs 3.a,3.b,3.c. and 3.d. the meshing has 3 domains Metal , Fluid (water) and air namely. A 1m spherical domain is used for the Air surrounding the metal bowl. Acoustic resonance modes simulations are carried out under both empty and partly filled conditions.

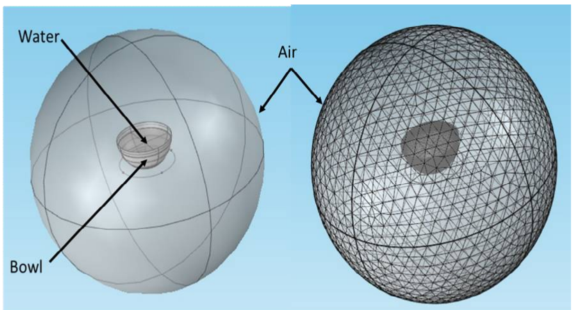

Fig 3.a Air, Water and Bowl   Fig3.b Mesh of the Air Domain

IV. EXPERIMENT SETUP AND RESULTS

The measurement arrangement is setup as shown in Fig 5.0. A multimeter with a thermocouple attached to the bowl is used for temperature measurement. The bowl is rubbed for the excitation and a microphone is used for audio recording at a 44.1kHz sampling rate. The following Fig 4.a shows a typical time domain signal of the exited bowl. The rubbing excitation excites the beats frequency modes as well as growing and followed by a degeneration phases after the rubbing stopped with respect to time.

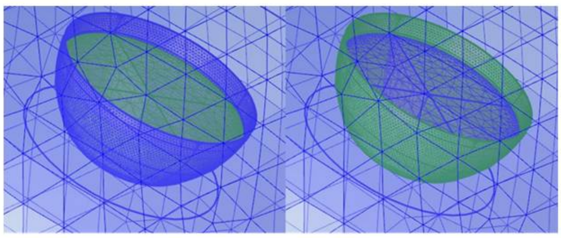

Fig 3.c Mesh of the Water Domain   Fig3.d Mesh of the Bowl Domain

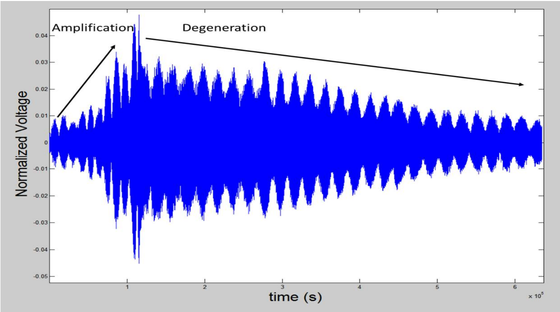

Fig 4.a Normalized Voltage vs time (s) Sampled at 44kHz

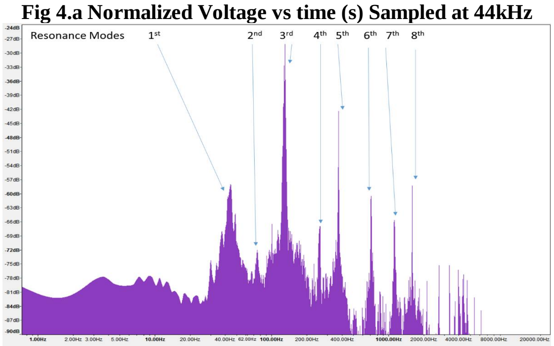

Fig 4.b Power Spectral Density Sampled at 44kHz

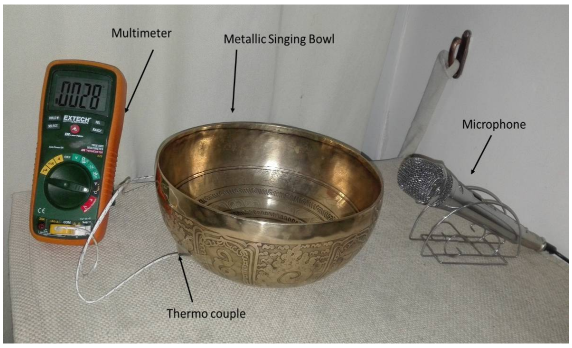

Fig 5.0 Experiment Setup

A typical Frequency domain power spectral density characteristics of the bowl is shown on the Fig 4.b. Only the first 8 resonance modes are considered for the discussion and higher order resonance were not considered. A typical Frequency and time spectrogram (PSD) of the Bowl is given in Fig 6.0. The frequency spectrums(first 8 Resonance modes Fig 4.b) are measured with empty bowl, partly filled bowled with 5cm,7cm and 9cm filled with water(total max eight is H=10cm). At each water level the frequency spectrums are measured for temperatures from 5^0C to 40^0C.

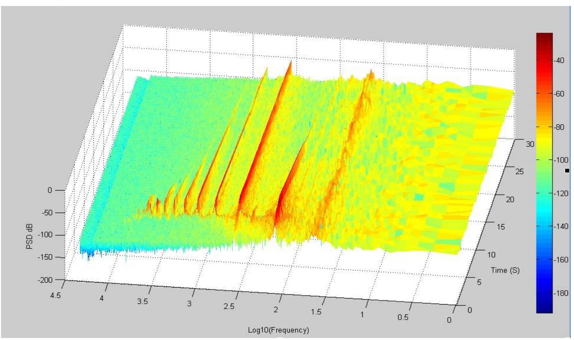

Fig 6.0 Spectrogram of Singing Bowl with rubbing excitation

The following Figures Fig 7.a-Fig 7.h show the measured frequency variations due to the Temperature and the water height. The resonance mode Frequency (F_n) is assumed to be a linear function of water height (h) and temperature (T) given by the following equation 10. The Coefficients are found by using the multiple linear regression and the results shown 11.1-11.8 equations.

$$F_n = C + Fh + GT \quad ----(10)$$

Where F,G and C are constants.

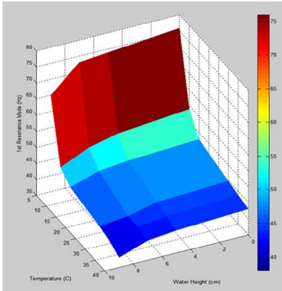

Fig 7.a 1st Resonance Mode

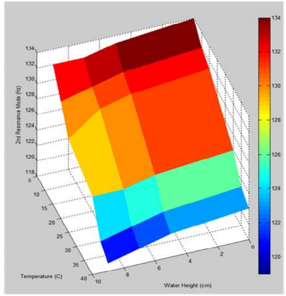

Fig 7.b 2nd Resonance Mode

$$F_1 = 66.8358 - 0.6136h - 0.644T \quad —(11.1)$$

$$F_2 = 135.9812 - 0.324h - 0.3458T \quad --(11.2)$$

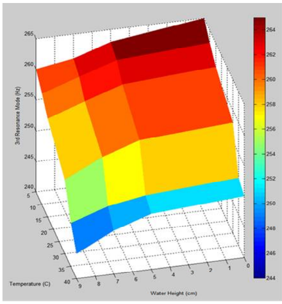

Fig 7.c 3rd Resonance Mode

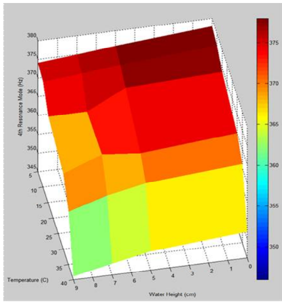

Fig 7.d 4th Resonance Mode

$$F_3 = 268.0657 - 0.3445h - 0.4373T \quad --(11.3)$$

$$F_4 = 383.6969 - 0.5736h - 0.6157T \quad --(11.4)$$

The following Figure 8.0 show the simulation results of the resonance modes. When m is not zero higher resonance modes have complicated geometrical displacement modes with faraday waves shown in Figure 9.0.

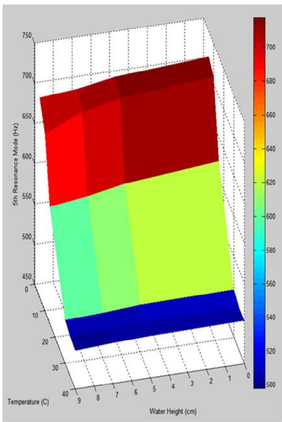

Fig 7.e 5th Resonance Mode

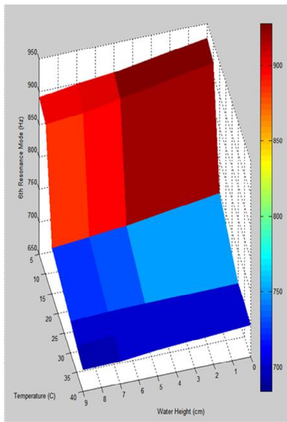

Fig 7.f 6th Resonance Mode

$$F_5 = 741.4023 - 1.811h - 6.4898T - -(11.5)$$
$$F_6 = 924.1942 - 2.5363h - 6.2687T - -(11.6)$$

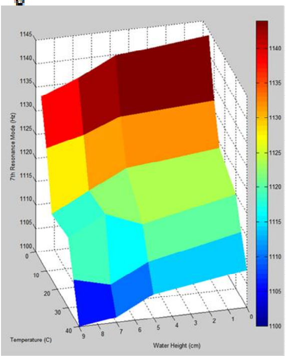

Fig 7.g 7th Resonance Mode

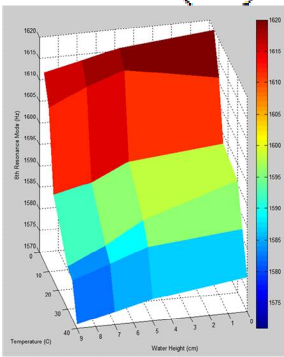

Fig 7.h 8th Resonance Mode

$$F_7 = 1143.5 - 0.8h - 0.9T - -(11.7)$$
$$F_8 = 1624 - 0.9h - 1.1 - -(11.8)$$

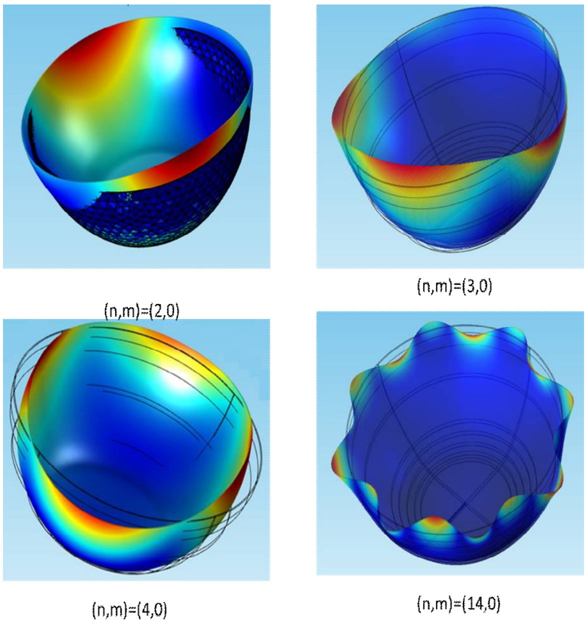

Fig 8.0 Resonance Modes where m=0

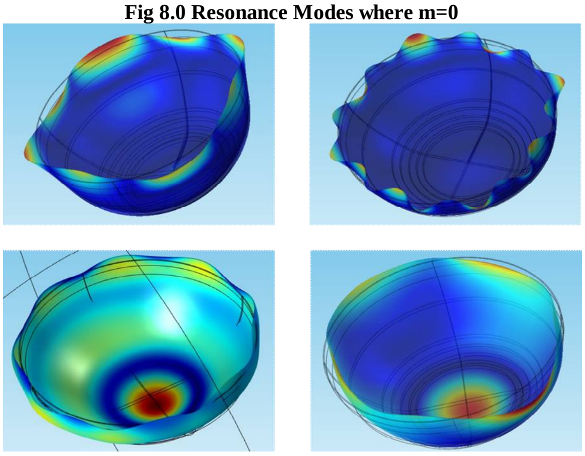

Fig 9.0 Resonance Modes with m≠0

DISCUSSION

The multiple linear regression Equations 11.1-11.8 has negative confidents for the water height and the Temperature therefore both factors having negative correlation towards the resonance mode frequency. The analytical model and the simulated numerical solutions considering the nonlinear effects also predicted reduction of the resonance mode frequency with the increase of Temperature and the Water height. Also all the multiple regression equations showed higher magnitudes for the temperature coefficients than the water height coefficients respectively. It is also observed that the resonance mode frequency shift is significant at lower modes and higher modes but less significant in the middle Resonance modes. The recorded audio track shows Beat frequencies. The beats may have been developed due to the nonlinear material properties of the bowl and the asymmetry of the bowl dimensions. Therefore it is very important to maintain the Metallic singing bowl at a particular temperature in order to generate the required resonance frequency modes. The author would like thank the Ven Samadhi Sukhino for providing the Tibetan singing bowl for the experiment.

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