GAIN SCHEDULING CONTROL WITH MULTI-LOOP PID FOR 2-DOF ARM ROBOT TRAJECTORY CONTROL

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Abstract— High accuracy trajectory tracking with a suitable rate of change in velocity is a very challenging topic in direct drive robot control. This challenge is due to the nonlinearities and input couplings present in the dynamics of robotic arms. In this paper, a 2 DOF robotic arm has been controlled by a multi-loop PID gain scheduling controller for a specific trajectory input. A nonlinear dynamic model of the manipulator has been obtained. A method of linearization was used for obtaining a linearized model for each set of individual operational points along the trajectory. A new proposed approach for merging between the Linearized models is introduced based on a weighting technique. A comparison between the output behaviour of the nonlinear model and the linearized model with the developed weighting technique has been carried out. A multi-loop PID controller has been designed for each individual linearized model as a MIMO plant. Then, the proposed controller was applied on the nonlinear plant using the weighting technique approach. The results have been compared at different trajectory inputs to guarantee the robustness and performance of the controller. The proposed approach has shown a simple design and good results over the other previous researches.

Keywords— Nonlinear Dynamic, Linearization, Gain Scheduling Control, PID, Weighting Technique, Fuzzy Logic.

I. INTRODUCTION

The serial robotic manipulators research is an essential part of continual activities of new manufacturing organizations overall the world. Serial manipulators play an important and extensive performance in the industrial revolution in those organizations. Studying of the serial robots, dynamic theories without concerning the forces produced at its end-effector is still representing a challenge for the researchers who using any types of the manipulators. The important point of dynamic studies of the manipulator is to grasp the nature and magnitude of forces acting, behaviour robustness and power requirements[1]. Tracking control for nonlinear systems with unknown disturbance and difference type of trajectory is a challenging problem. To reach good tracking path under uncertainties, one usually needs to combine all of the following mechanisms in the control design: adaptation, feed-forward, and high-gain[2].Manipulator control purpose is to conserve the dynamic response of a robot in accordance with pre-specified objectives. The dynamic performance of a manipulator directly depends on the robustness of the control algorithms and the dynamic model of the robot. Most of the current robot arm control designs treat each joint of the manipulator as a simpler linear servomechanism with simple controller like Independent Joint Control (IJC), PD, or PID controllers. In this approach, the nonlinear, coupled and time-varying dynamics of the mechanical part of the robot manipulator system have usually been completely ignored, or assumed as disturbance. However, when the links are moving simultaneously and at high speed, the nonlinear coupling effects and the interaction forces between the manipulator links may decrease the performance of the overall system and increase the tracking error[3]. One of the most efficient controller deals with nonlinear systems is Variable Structure Control (VSC) with sliding mode control (SMC). VSC has been developed into a general design method being examined for a wide spectrum of system types including nonlinear systems, MIMO systems, discrete-time model, and infinite-dimension systems[4]. In addition to these studies, fuzzy logic controllers have been integrated with the SMC[5][6], where fuzzy logic is used for adaption of the controller parameters and to tune the discontinuous control gain of the conventional SMC.Gain scheduling is a design methodology, which has been used in many real nonlinear applications (e.g. jet engines, submarines, and aircraft) with complex design for a nonlinear controller. Gain scheduling ideais to select several operating points, which cover the range of the plant's dynamics. Then, construct a linear time-invariant approximation plant at every operating point individually followed by designing a linear compensator for each linearized plant. Between operating points, the gains of the compensators interpolated, or scheduled resulting in a global compensator. Gain scheduling designs have been guided by heuristic rules-of-thumb. The two most essential guidelines are: 1) the scheduling variable should vary slowly; and 2) the scheduling variable should capture the plant's nonlinearities. Nonlinear gain scheduling has two types: 1) a nonlinear plant scheduling on a reference trajectory; and 2) a nonlinear scheduling on the plant output[7],[8]. Each linearized model is considered as multivariable or multi-input multi-output (MIMO) system. Therefore multi-loop PID is one of the best controllers due to their satisfactory performance along with their simple, failure tolerant, and easy to understand structure. MIMO systems have complex interactive natures, which make the proper tuning of multi-loop PID controller quite difficult. For this reason, the number of applicable tuning methods is relatively limited. A computational method is as shown in[9].

In this paper, Gain scheduling control integrated with multi-loop PID is discussed and implemented to reach the best performance on a reference trajectory. The developed control design is intend to be simple compared with the others nonlinear controllers, which need complex calculations to construct. A new weighting technique approach is used in collecting the output of linearized plants. This guarantees the robustness of the linearized model in replacing the nonlinear model.

II. DYNAMIC MODEL

2.1. Nonlinear Mathematical Equations

The dynamical analysis of the robot shows a relation between the joint torques applied by the actuators and the position, velocity and acceleration of the robot arm with respect to time. High nonlinearity is appears in the model differential equations that might make a challenge for accurate and robust controller design. Therefore, the 2-DOF arm robot in Fig.1 is good example to test performance of the controller.

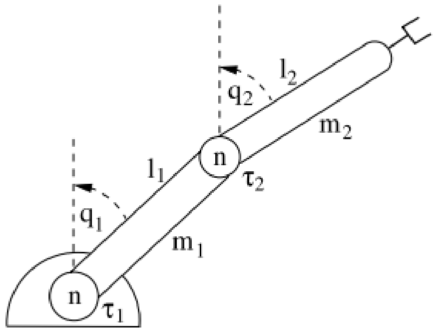

Fig.1.Robotmanipulator two degrees of freedom

Using the Euler-Lagrange Eq. 1 formulation, the rigid-body dynamic model of the two-link manipulator is obtained as:

$$\tau(t) = D\big(q(t)\big)\ddot{q}(t) + C(q,\dot{q}) + G(q) \quad (1)$$

$$\therefore q_1 = \theta_1$$

$$\therefore q_2 = \theta_2$$

Where $D(q)$ is the inertia matrix, $C(q,\dot{q})$ is the coriolis/centripetal matrix, $G(q)$ is the gravity vector, and T is the control input torque. The joint variable q is an n-vector containing the joint angles for revolute joints. The dynamic equation Eq. 2 of the 2-DOF robotic arm can be computed by:

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}(m_1+m_2)l_1^2 + m_2 l_2 + 3m_2 l_1 l_2\cos\theta_2 & m_2 l_2^2 + \frac{3}{2}m_2 l_1 l_2 \cos\theta_2\\ m_2 l_2^2 + m_2 l_1 l_2\cos\theta_2 & m_2 l_2^2\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} + \begin{bmatrix}-m_2 l_1 l_2\left(\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}\dot{\theta}_2^2\right)\sin\theta_2\\ \frac{1}{2}m_2 l_1 l_2 \dot{\theta}_1^2 \sin\theta_2\end{bmatrix} + \begin{bmatrix}\left(\frac{m_1}{2} + m_2\right) g l_1 \sin\theta_1 + m_2 g\frac{l_2}{2}\sin(\theta_1+\theta_2)\\ m_2 g \frac{l_2}{2}\sin(\theta_1+\theta_2)\end{bmatrix} \quad (2)$$

Where m_i is link mass l_i is link length, g is the gravity and $\theta, \dot{\theta}$ and $\ddot{\theta}$ respectively are the joint positions, velocities and accelerations[1].

Actuators model are computed to merge it with the dynamic model of the robot as:

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}K_{m1}\\ K_{m2}\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix} - \begin{bmatrix}B_{m1}.n^2\\ B_{m2}.n^2\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} - \begin{bmatrix}J_{m1}.n^2\\ J_{m2}.n^2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} \quad (3)$$

$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix} = \begin{bmatrix}\frac{1}{R_1}\\ \frac{1}{R_2}\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix} - \begin{bmatrix}\frac{L_1}{R_1}\\ \frac{L_2}{R_2}\end{bmatrix}\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix} - \begin{bmatrix}K_{e1}.n\\ K_{e2}.n\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} \quad (4)$$

where I_i is motor current, K_{m_i} is motors constant, $B_{m_i}$is friction/damping constant including links, $J_{m_i}$is motor inertia, K_{e_i} is back-emf constant, R_i is motor resistance, L_i is motor inductance, n is the gear box ratio, V_i is the input voltage to the motors.

Table 1: Shows the actual DC-motor parameters are used

Parameters	*Motor1*	*Motor2*
K_m[N-m/amp]	0.00356	0.0041
K_e [volt-sec/rad]	0.001	0.0011
J_m[N-m-sec^2]	7.02*10^{-6}	6.64*10^{-6}
R [ohms]	0.823	0.8
L [henries]	0.0009	0.0009
B_m [N-m-sec]	2.13*10^{-6}	1.956*10^{-6}

2.2. Linearization

Linearization procedures are introduced to build a simple model with the same or acceptable output behaviour around a certain trajectory compared to the real nonlinear model. Taylor series is used in linearizing the nonlinear model around each operating point in terms of 1st order system, which is the most famous and easiest method of linearization[10]. As shown in Fig. 2, θ_1 range from 0° to 70° degree which is scheduled every 10°degrees into 8 points, then θ_2 range from 0° to 60° degree which is scheduled every 10° degrees into 7 points for every single θ_1. The total operating points are about 56 linearized model cover a large usable workspace for the end-effector. The trajectory between every two successive scheduled points is assumed to be linear. The gain-scheduling technique is not ended by generating the linearized models; the point is how to manage between each linearized model to use the suitable one against a certain position and orientation.

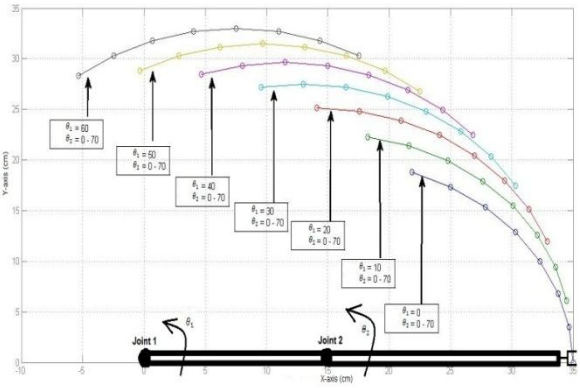

Fig.2. End-effector workspace trajectory points

Weighting technique is used to compute the output dynamic behaviour from the linearized models at any desired position and orientation. The idea of this technique is very simple and effective on calculating the dynamic behaviour output. All the models are ignored from computing the output only if the two models lie between certain θ_1 *and* θ_2 equivalent to the desired position and orientation according to the forward kinematics equations[1]. The technique introduces the notation of ignoring the linearized plant with unwanted output, which minimize the complicity of the model and reaching a faster response. By using the weighting technique between the only two models that describe the dynamic the behaviour of the robot at certain values is the easiest and robust method in linearizing nonlinear plant about known trajectory. Thus, the controller design of each linear time-invariant plant will act with good performance in correct and reaching an accurate position and orientation. The outputs of each two single models are weighted using this equation:

$$Y = \sum_{n=0}^{n=6} a_n . \left[\left(1 - \left(\frac{Y_n}{10} - n \right) \right) . \theta_{i,j} + \left(\left(\frac{Y_n}{10} - n \right) . \theta_{i,j+1} \right) \right]$$

$$a_n = 1 \text{at} Y = Y_n$$

$$a_n = 0 \text{ at} Y \neq Y_n (5)$$

Where Y is the total output behaviour of the linearized models, $\theta_{i,j}$ describe each linearized model individually where i for joint 1 (θ_1) &j for joint 2 (θ_2).The linearized models are reconstructed with different scheduling steps of 15° and 20° degrees for the same range of trajectory. Error between linearized model and nonlinear one is increase by increasing the step (delta) between points along trajectory as shown in Fig. 3 and 4. The robustness and performance of the linearized model with weighting technique and nonlinear model is compared by applying step input for both of linear and nonlinear models as illustrated in Fig. 5 and 6.

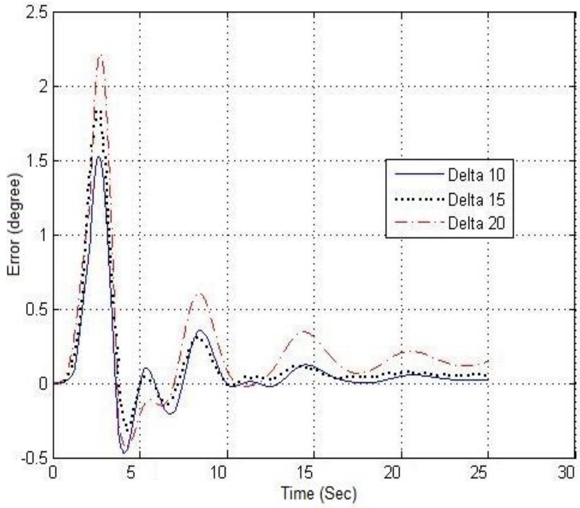

Fig.3. Position error of joint 1

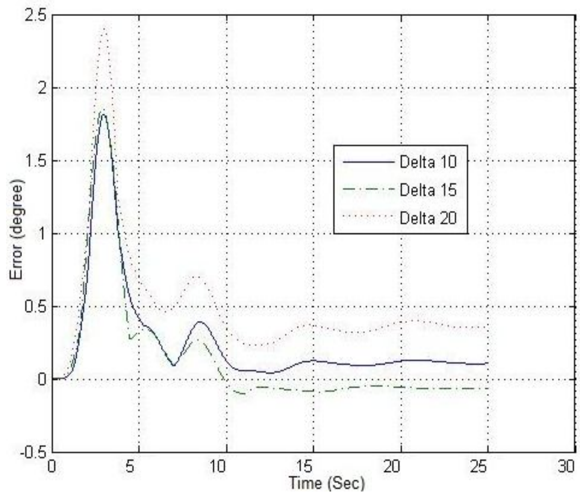

Fig.4. Position error of joint 2

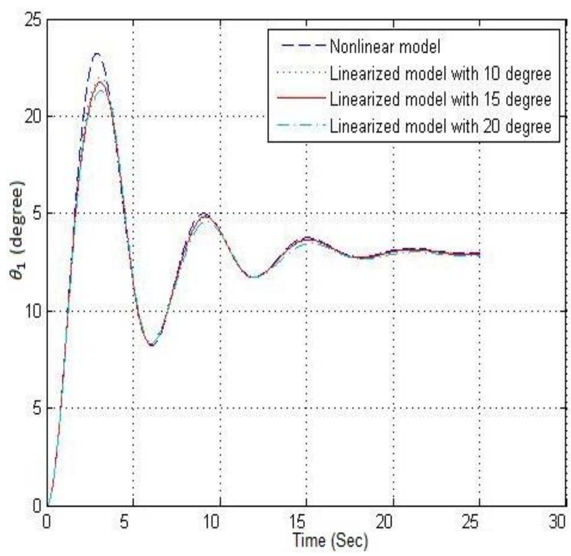

Fig.5. Step input at joint 1

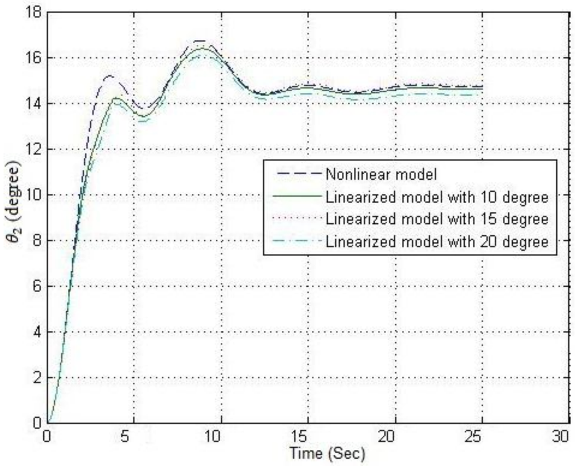

Fig.6. Step input at joint 2

In this part, a nonlinear dynamic model of the 2-DOF arm and the actuator model have been introduced. A certain trajectory has scheduled every $10°, 15°$and$20°$. The increase in difference between each scheduled point, the increase in error between the linearized dynamic output and the nonlinear dynamic behavior. Linearization about each operating point along the trajectory has been discussed with the weighting technique, which guarantees the best and robust performance of the linearized dynamic behaviour output.

III. GAIN SCHEDULING CONTROL

Gain scheduling is popular approaches used with nonlinear control design and has been widely and successfully applied in fields ranging from aerospace to process control. Although a wide variety of control methods are often described as "gain-scheduling" approaches, these are usually related by a partition type of design procedure whereby the nonlinear control design task is decomposed into a number of linear sub-systems. This division approach is the source of much of the popularity of gain-scheduling methods since it enables a well-established linear design methods to be applied over nonlinear problems[11]. The procedures of designing local gain-scheduling controller based on linear time-invariant approximations plants have been discussed. Linearization around the operating points and verifying them with the nonlinear model in the previous section is the first step to guarantee a robust stability and robust performance of the gain-scheduling controller. For every individual linearized model a feedback system is added with a compensator. Multi-loop PID[12] is designed as a feedback compensator to reach the requirements of gain margin and phase for every single linearized model with separate design. The tuning process of the PID gains is selected using Robust control Matlab Toolbox for MIMO systems.Consider each linearized plant has two inputs and two outputs, so that Eq. 8 $\{G_{ij}\}$ is an open-loop transfer function 2×2 matrixes that represent the dynamic of each plant, and it given as:

$$Y(s) = G(s)U(s) + D(s) \quad (6)$$

Where $Y(s), U(s)$, and $D(s)$ are the output, input, and disturbance vectors.

$$G_{i,j}(s) = \begin{bmatrix} g_{1,1}(s) & g_{1,2}(s) \\ g_{2,1}(s) & g_{2,2}(s) \end{bmatrix} \quad (7)$$

$$i = (0°, 10°, 20°, 30°, 40°, 50°, 60°)$$
$$j = (0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°)$$

Where i, j describe the trajectory operating points.

$$G_{i,j} = \frac{1}{S^6 + a_{00}S^5 + a_{01}s^4 + a_{02}S^3 + a_{03}S^2 + a_{04}S + a_{05}} \times \begin{bmatrix} b_{00}S^3 + b_{01}s^2 + b_{02}S + b_{03} & c_{00}S^3 + c_{01}s^2 + c_{02}S + c_{03} \\ d_{00}S^3 + d_{01}s^2 + d_{02}S + d_{03} & e_{00}S^3 + e_{01}s^2 + e_{02}S + e_{03} \end{bmatrix} \quad (8)$$

The end-effector trajectory has been scheduled into 56 operating point, each point has a plant transfer function describes the linear time-invariant behaviour around this point$\{G_{i,j}\}$. The linearized models which describe the robot arm behaviour around the trajectory in appendix A. The closed loop block diagram in Fig.7shows the decoupling between the transfer functions, input, output and controller. Two PID controllers (C1 and C2) are used, one for joint 1 and the other one for joint 2. The controllers have been tuned under certain gain margin and gain phase to reach the required time response and steady state error.

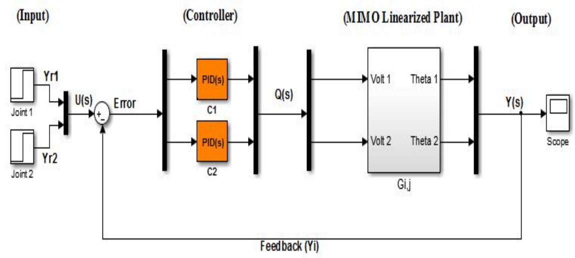

Fig.7. Closed loop MIMO PID controller

PID gains (K_p, K_i, K_d) which used with both controllers are tuned using some iteration by Robust Matlab Toolbox with certain algorithm, which gives an accepted and reasonable output behavior. This procedure repeated for each linearized model $\{G_{i,j}\}$ to design a specific controller, then used the same technique of weighting between the controllers, which is discussed in the previous section.

$$\begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix} = \begin{bmatrix} C_1(Y_{r1}(s) - Y_1(s)) \\ C_2(Y_{r2}(s) - Y_2(s)) \end{bmatrix} \quad (9)$$

$$C_i = K_{P_i} + \frac{K_{I_i}}{s} + K_{D_i}s$$

Where Q_i are the control signal, Y_{ri} the desired trajectory, Y_i the process output, and C_i controller gains.
After applying the desired trajectory path as an input for both joints, results are shown for joint 1 in Fig.8and for joint 2 in Fig. 9. The results show the robustness in stability and performance of the output relative to the input trajectory with a good agreement between the curves.

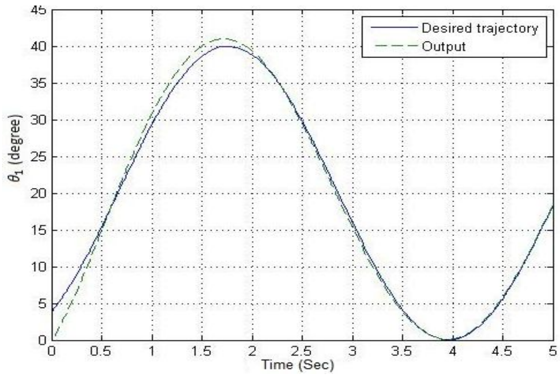

Fig.8. Tracking output of joint 1

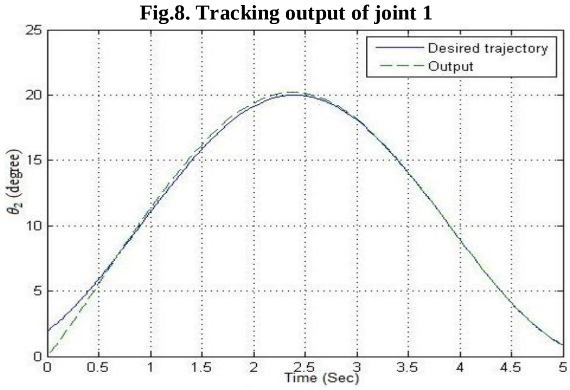

Fig.9. Tracking output of joint 2

IV. VERIFYING RESULTS

In this section, the output of the end-effector around a certain trajectory using the gain-scheduling technique is compared with previous article using fuzzy logic (FLC) with PSO optimization[13]. Trajectory input in both controllers is approximately the same to acquire a reasonable comparison as shown in Fig. 10 and 11.

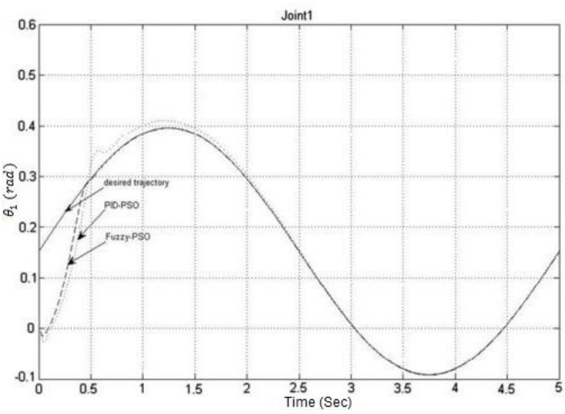

Fig.10. FLC output of joint 1

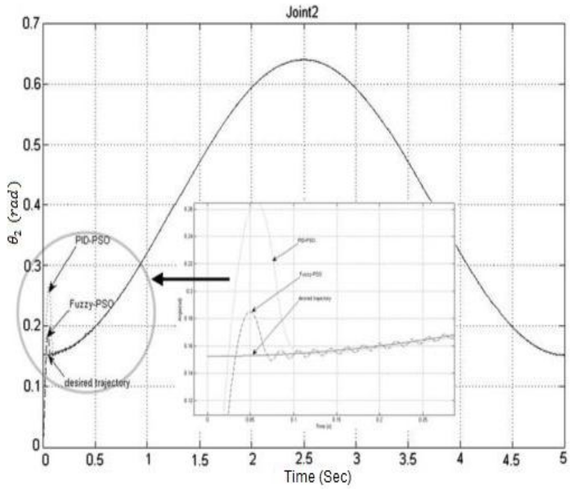

Fig.11. FLC output of joint 2

The comparing results verified that the gain-scheduling control with the technique of weighting and merging multi-loop PID has a high performance and robust behaviour in tracking a certain trajectory with simplicity in design relative to the other FLC techniques. As shown in Fig. 8 and 10 for joint 1, gain-scheduling controller has a fast response from the zero position with a small maximum overshoot (MOS), while FLC records a late response from zero position. Joint 2 has a fast response with FLC, but with high MOS and steady state error, which may affect the position control accuracy due to the arm inertia. However, gain-scheduling controller records a slow response at starting, which can keep the end-effector more stable on starting from zero position and avoid any fluctuations on link 2, as shown in Fig. 9 and 11.

CONCLUSIONS

Nonlinear system controller design is a common problem in dynamic control. Many techniques have been proposed every day to achieve the simplicity and the robust performance in control design. This paper introduces a simple technique with robust performance in trajectory tracking control. Gain-scheduling technique was used with some changes in scheduling calculation, which leads to a linear model with high performance comparing to the nonlinear model. Multi-loop PID has been merged with the gain scheduling as a compensator, which forces the end-effector to track the trajectory with an acceptable steady state error. The results of the simulation show the robust rendering of the gain-scheduling technique compared by the other algorithms.The simplicity of gain-scheduling design with weighting technique merged by multi-loop PID leads to design a simple nonlinear controller for two inputs and two outputs (MIMO) with guarantee of robust stability and high performance in tracking any trajectory path.

APPENDIX A LINEARIZED PARAMETERS

I. For $\theta_1 = 0^\circ \in 0^\circ \le \theta_2 < 60^\circ$

	$G_{10,0}$	$G_{10,10}$	$G_{10,20}$	$G_{10,30}$	$G_{10,40}$	$G_{10,50}$	$G_{10,60}$
a_{00}	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4
a_{01}	838389	838389.9	838391	838394	838398	838403	838408
$a_{02}.10^6$	1.55	1.55	1.55	1.55	1.55	1.55	1.56
$a_{03}.10^6$	5.48	5.37	5.22	5	4.72	4.41	4.05
$a_{04}.10^6$	4.57	4.48	4.32	4.11	3.85	3.54	3.2
$a_{05}.10^6$	3.7	3.53	3.26	2.9	2.46	1.95	1.37
b_{00}	3033	3035	3040.7	3050	3063	3079	3098
$b_{01}.10^6$	2.77	2.77	2.78	2.78	2.8	2.81	2.83
$b_{02}.10^6$	2.66	2.66	2.67	2.68	2.7	2.71	2.72
$b_{03}.10^6$	4.24	4.06	3.76	3.36	2.86	2.27	1.6
c_{00}	-136	-135	-132	-127	-120.3	-112	-102.2
$-c_{01}$	124371	123449	120703	116198	110041	102379	93404
$-c_{02}$	4637.5	4458.4	4165.3	3762.6	3256.5	2655.1	1968.8
$c_{03}.10^6$	-4.24	-4.07	-3.8	-3.44	-2.97	-2.42	-1.8
d_{00}	-136	-135	-132	-127	-120.3	-112	-102.2
$-d_{01}$	124371	123449	120703	116198	110041	102379	93404
d_{02}	-4637.5	-4427.8	-4088.4	3627.6	3056.7	2390.2	1645
$d_{03}.10^6$	-4.24	-4.05	-3.74	-3.3	-2.79	-2.18	-1.5
e_{00}	3312	3312	3311.7	3311.2	3310.7	3310	3309.3
$e_{01}.10^6$	3.02	3.02	3.03	3.02	3.02	3.02	3.02
$e_{02}.10^6$	2.67	2.67	2.68	2.68	2.7	2.71	2.72
$e_{03}.10^7$	1.29	1.28	1.25	1.21	1.16	1.1	1.04

II. For $\theta_1 = 10^\circ \in 0^\circ \le \theta_2 < 60^\circ$

	$G_{0,0}$	$G_{0,10}$	$G_{0,20}$	$G_{0,30}$	$G_{0,40}$	$G_{0,50}$	$G_{0,60}$
a_{00}	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4
a_{01}	838389	838389	838391	838394	838398	838403	838409
$a_{02}.10^6$	1.5	1.5	1.5	1.55	1.55	1.55	1.56
$a_{03}.10^6$	5.5	5.5	5.4	5.26	5.05	4.78	4.47
$a_{04}.10^6$	4.6	4.6	4.5	4.37	4.16	3.9	3.6
$a_{05}.10^6$	3.8	3.7	3.6	3.32	2.95	2.5	1.97
b_{00}	3033	3034.9	3040.7	3050	3063	3079	3098
$b_{01}.10^6$	2.7	2.7	2.7	2.78	2.8	2.81	2.83
$b_{02}.10^6$	2.6	2.6	2.6	2.67	2.69	2.7	2.72
$b_{03}.10^6$	4.3	4.2	4	3.77	3.37	2.86	2.27
c_{00}	-136	-135	-132	-127	-120.3	-112	-102
$-c_{01}$	124371	123448	120703	116198	110040	102379	93404
$-c_{02}$	4709	4649.6	4471	4177.8	3771	3256.7	2405
$c_{03}.10^6$	-4.3	-4.2	-4	-3.82	-3.33	-2.97	2.2
d_{00}	-136	-135	-132	-127	-120.3	-112	-102
$-d_{01}$	124371	123448	120703	116198	110040	102379	93404
d_{02}	-4709	-4640.4	-4436.1	-4101	-3643	-3072.7	-2405
$d_{03}.10^6$	-4.3	-4.2	-4	-3.82	-3.33	-2.8	-2.2
e_{00}	3312	3312	3311.7	3311.2	3310.7	3310	3309
$e_{01}.10^6$	3	3	3	3.02	3.02	3.02	3.02
$e_{02}.10^6$	2.6	2.6	2.6	2.68	2.7	2.71	2.73
$e_{03}.10^7$	1.3	1.3	1.3	1.26	1.23	1.18	1.12

III. For □□ = □□° ∈ □° ≤ □□ < □□°

	$G_{20,0}$	$G_{20,10}$	$G_{20,20}$	$G_{20,30}$	$G_{20,40}$	$G_{20,50}$	$G_{20,60}$
a_{00}	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4
a_{01}	838388	838389	838391	838393	838397	838402	838408
$a_{02}.10^6$	1.55	1.55	1.55	1.55	1.55	1.55	1.56
$a_{03}.10^6$	5.26	5.09	4.87	4.6	4.27	3.92	3.54
$a_{04}.10^6$	4.36	4.2	4	3.73	3.41	3.07	2.7
$a_{05}.10^6$	3.36	3.11	2.76	2.34	1.85	1.31	0.73
b_{00}	3033	3035	3040.7	3050	3063	3079	3098
$b_{01}.10^6$	2.77	2.77	2.78	2.79	2.8	2.81	2.83
$b_{02}.10^6$	2.66	2.66	2.67	2.67	2.68	2.7	2.71
$b_{03}.10^6$	4.04	3.75	3.34	2.85	2.26	1.61	0.9
c_{00}	-136	-135	-132	-127	-120.3	-112	-102
$-c_{01}$	124371	123448	120703	116198	110040	102380	93404.4
$-c_{02}$	4425	4131.7	3732	3233	2643	1973	1234.7
$c_{03}.10^6$	-4.04	-3.77	-3.41	-2.95	-2.41	-1.8	-1.13
d_{00}	-136	-135	-132	-127	-120.3	-112	-102
$-d_{01}$	124371	123448	120703	116198	110041	102380	93404.4
d_{02}	-4425	-4080.7	-3616.4	-3044	-2378	-1635	-835.2
$d_{03}.10^6$	-4.04	-3.73	-3.3	-2.78	-2.17	-1.5	-0.76
e_{00}	3312.1	3312	3311.7	3311.2	3310.7	3310	3309.3
$e_{01}.10^6$	3.03	3.03	3.02	3.02	3.02	3.02	3.02
$e_{02}.10^6$	2.67	2.67	2.68	2.68	2.7	2.7	2.72
$e_{03}.10^7$	1.23	1.2	1.16	1.11	1.06	1	0.92

IV. For □□ = □□° ∈ □° ≤ □□ < □□°

	$G_{40,0}$	$G_{40,10}$	$G_{40,20}$	$G_{40,30}$	$G_{40,40}$	$G_{40,50}$	$G_{40,60}$
a_{00}	829.4	829.4	829.4	829.4	829.4	829.4	829.4
a_{01}	838397	38388.7	838389	838392	838396	838400	838406
$a_{02}.10^6$	1.54	1.54	1.54	1.55	1.55	1.55	1.55
$a_{03}.10^6$	4.42	4.13	3.81	3.45	3.07	2.67	2.27
$a_{04}.10^6$	3.56	3.28	2.97	2.63	2.26	1.87	1.49
$a_{05}.10^6$	2.23	1.89	1.5	1.05	0.6	0.11	-0.36
b_{00}	3033	3035	3040.7	3050	3063	3079	3098
$b_{01}.10^6$	2.77	2.77	2.78	2.78	2.8	2.81	2.83
$b_{02}.10^6$	2.66	2.66	2.67	2.67	2.68	2.7	2.71
$b_{03}.10^6$	3.3	2.8	2.22	1.58	0.9	0.17	-0.57
c_{00}	-136	-135	-132	-127	-120	-112	-102
$-c_{01}$	24371	23448	20703	16198	10041	02379	93404.4
$-c_{02}$	607.3	115.4	542	898.5	196.7	51	22.6
$c_{03}.10^6$	-3.3	-2.84	-2.32	-1.73	-1.1	-0.41	0.29
d_{00}	-136	-135	-132	-132	-120	-112	-102
$-d_{01}$	124371	123448	20703	16198	110041	102379	93404.4
d_{02}	-3607.3	-3029	-2360.4	-1619.6	-825.7	0	835.2
$d_{03}.10^6$	-3.3	-2.76	-2.32	-1.48	-0.75	0	0.76
e_{00}	3312	3312	3311.7	3311.2	3310.7	3310	3309.3
$e_{01}.10^6$	3.02	3.02	3.02	3.02	3.02	3.02	3.02
$e_{02}.10^6$	2.67	2.67	2.67	2.68	2.7	2.7	2.72
$e_{03}.10^7$	1.01	0.95	0.89	0.83	0.76	0.69	0.61

V. For □□ = □□° ∈ □° ≤ □□ < □□°

	$G_{30,0}$	$G_{30,10}$	$G_{30,20}$	G_{3030}	$G_{30,40}$	$G_{30,50}$	$G_{30,60}$
a_{00}	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4
a_{01}	838388	838388.7	838390	838393	838396	838401	838407
$a_{02}.10^6$	1.54	1.54	1.55	1.55	1.55	1.55	1.56
$a_{03}.10^6$	4.9	4.67	4.4	4.07	3.72	3.34	2.94
$a_{04}.10^6$	4.02	3.8	3.53	3.22	2.88	2.51	2.12
$a_{05}.10^6$	2.86	2.54	2.15	1.7	1.2	0.67	0.13
b_{30}	3033	3035	3040.7	3050	3063	3079	3098
$b_{01}.10^6$	2.77	2.77	2.78	2.79	2.8	2.81	2.83
$b_{02}.10^6$	2.66	2.66	3.67	2.67	2.68	2.7	2.71
$b_{03}.10^6$	3.72	3.32	2.83	2.25	1.6	0.9	0.17
c_{00}	-136	-135	-132	-127	-120	-112	-102
$-c_{01}$	124371	123449	120703	116198	110041	102379	93404
$-c_{02}$	4078	3579	3185.6	2605.4	1950	1230.5	463.1
$c_{03}.10^6$	-3.73	-3.36	-2.9	-2.38	-1.48	-1.12	-0.42
d_{00}	-136	-135	-132	-127	-120	-112	-102.1
$-d_{01}$	124371	123449	120703	116198	110041	102379	93404
d_{02}	-4078	-3609	-3034.5	-2367.7	-1626.4	-830.1	0
$d_{03}.10^6$	-3.73	-3.3	-2.77	-2.15	-1.48	-0.76	0
e_{30}	3312	3312	3311.7	3311	3310.6	3310	3309.3
$e_{31}.10^6$	3.03	3.03	3.02	3.02	3.02	3.02	3.02
$e_{32}.10^6$	2.67	2.67	2.67	2.68	2.7	2.71	2.72
$e_{33}.10^7$	1.14	1.1	1.04	0.98	0.92	0.85	0.78

VI. For □□ = □□° ∈ □° ≤ □□ < □□°

	$G_{50,0}$	$G_{50,10}$	$G_{50,20}$	$G_{50,30}$	$G_{50,40}$	$G_{50,50}$	$G_{50,60}$
a_{00}	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4
a_{01}	838387	838387.3	838388	838391	838395	838400	838405
$a_{02}.10^6$	1.54	1.54	1.54	1.54	1.55	1.55	1.55
$a_{03}.10^6$	3.82	3.5	3.13	2.75	2.35	1.95	1.56
$a_{04}.10^6$	2.98	2.67	2.32	1.9	1.57	1.18	0.8
$a_{05}.10^6$	1.57	1.23	0.87	0.48	0.089	-0.3	-0.7
b_{00}	3033	3035	3040.7	3050	3063	3079	3098
$b_{01}.10^6$	2.77	2.77	2.78	2.78	2.8	2.81	2.83
$b_{02}.10^6$	2.66	2.66	2.67	2.67	2.68	2.7	2.71
$b_{03}.10^6$	2.76	2.19	1.55	0.87	0.15	-0.57	-1.29
c_{00}	-136	-135	-132	-127	-120.3	-112	-102.2
$-c_{01}$	124371	123448.7	120703	116198	110041	102379	93404.4
$-c_{02}$	3027	2456.7	-1821.6	-1134	-407.4	42.4	1098.5
$c_{03}.10^6$	-2.76	-2.24	-1.66	-1.03	-0.37	0.31	1
d_{00}	-136	-135	-132	-127	-120.3	-112	-102.2
$-d_{01}$	124371	123448.7	120703	116198	110041	102379	93404.4
d_{02}	-3027	-2356	-1614.6	-822.3	0	830.1	1645
$d_{03}.10^6$	-2.76	-2.15	-1.47	-0.75	0	0.76	1.5
e_{00}	3312	3312	3311.7	3311.2	3310	3310	3309.3
$e_{01}.10^6$	3.02	3.02	3.02	3.02	3.02	3.02	3.02
$e_{02}.10^6$	2.67	2.67	2.67	2.68	2.7	2.7	2.72
$e_{03}.10^7$	0.84	0.78	0.72	0.65	0.57	0.5	0.43

VII. For □□ = □□° ∈ □° ≤ □□ < □□°

	$G_{60,0}$	$G_{60,10}$	$G_{60,20}$	$G_{60,30}$	$G_{60,40}$	$G_{60,50}$	$G_{60,60}$
a_{00}	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4	1829.4
a_{01}	838386	838386.4	838388	838390	838394	838399	838405
$a_{02}.10^6$	1.54	1.54	1.54	1.54	1.54	1.55	1.55
$a_{03}.10^6$	3.13	2.76	2.38	1.98	1.58	1.19	0.81
$a_{04}.10^6$	2.32	1.97	1.6	1.22	0.83	0.45	0.93
$a_{05}.10^6$	0.95	0.66	0.36	0.057	-0.24	-0.53	-0.81
b_{00}	3033	3035	3040.7	3050	3062	3079	3098
$b_{01}.10^6$	2.77	2.77	2.78	2.79	2.8	2.81	2.83
$b_{02}.10^6$	2.66	2.66	2.66	2.67	2.68	2.7	2.71
$b_{03}.10^6$	2.15	1.51	0.83	0.13	-0.58	-1.3	-1.97
c_{00}	-136	-135	-132	-127	-120.3	-112	-102
$-c_{01}$	124371	123448	120703	116198	110041	102379	93404
$-c_{02}$	2354.5	1723.4	1046	335	394	1125.3	1841
$c_{03}.10^6$	-2.15	-1.57	-0.95	-0.3	0.36	1.02	1.68
d_{00}	-136	-135	-132	-127	-120	-112	-102
$-d_{01}$	124371	123448	120703	116198	110041	102379	93404
d_{02}	-2354.5	-1611.6	-820	0	394	1635	2405
$d_{03}.10^6$	2.15	1.47	0.75	0	0.36	1.5	2.2
e_{00}	3312	3311.9	3311.7	3311.2	3310.6	3310	3309.3
$e_{01}.10^6$	3.02	3.02	3.02	3.02	3.02	3.02	3.02
$e_{02}.10^6$	2.66	2.66	2.67	2.68	2.69	2.7	2.72
$e_{03}.10^7$	0.66	0.59	0.52	0.44	0.37	0.3	0.23

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